

# SHEAR STRENGTH OF SOILS

## (PART-1)

Materi Disiapkan Oleh :

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# Geotechnical Engineering Concerns and Applications Related to Shear Strength of Soils:

- Foundation Engineering
- Slope Stability
- Tunneling and Deep Excavation
- Dynamic Problems



## Particular Emphasis:

**=> Total and Effective Stress Approach for Solution of  
Stability Problems in Geotechnical Engineering**



# Introduction

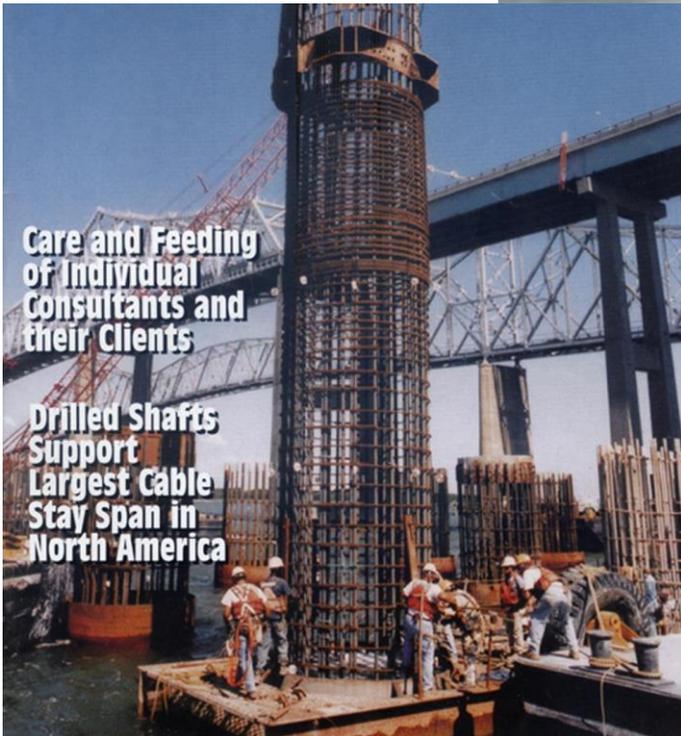


Geotechnical Concern and  
Practical Challenges



## Geotechnical Concern and Practical Challenges





Geotechnical Concern and  
Practical Challenges

# Land Reclamation



Geotechnical Concern and  
Practical Challenges

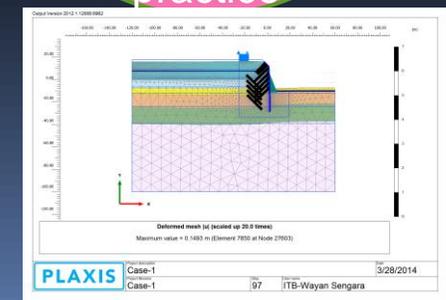
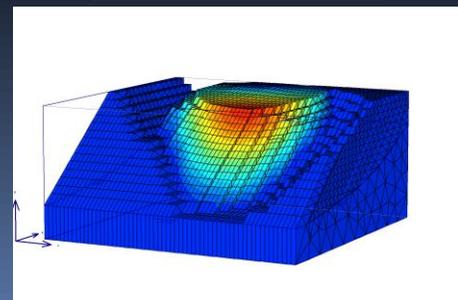
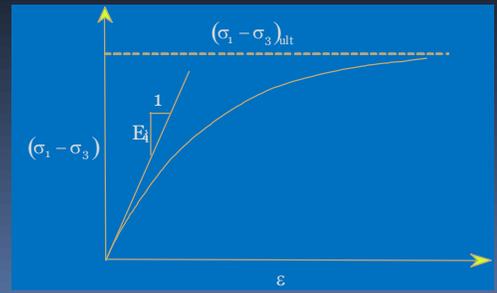
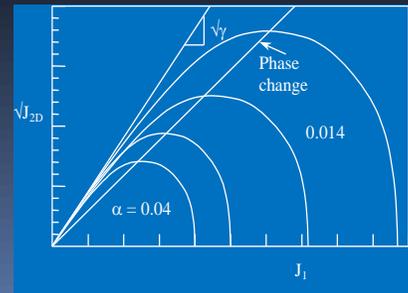
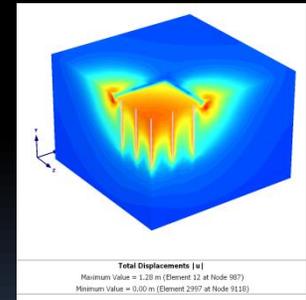
# (Geotechnical Modeling and Soil-Structure Interaction Analysis)

Knowledge of constitutive law of soils/  
Soil Plasticity  
→ Failure Criteria  
(*nonlinear, elasto-plastic, non-associative, time-dependent, etc*):  
MC; HS; SS

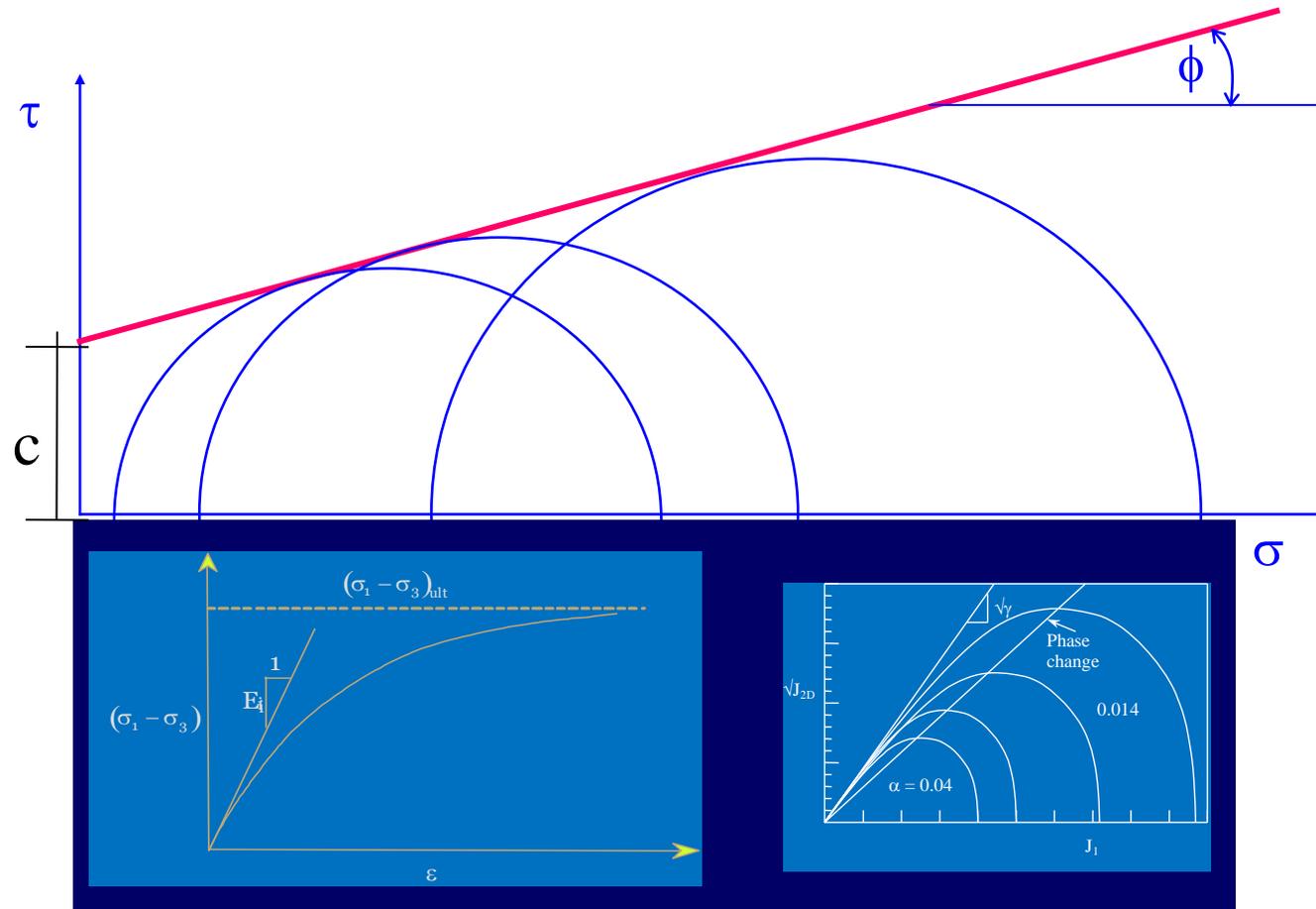
• Soil Modeling  
• Numerical Solution:  
(FEM)

Geotechnical Problem and SSI

Solution Complexity on Geotechnical Problem and SSI for research and practice



# NonLinear and Elasto-Plastic Stress-Strain Relationship and Mohr-Coulomb Failure Criteria



# Coarse- and Fine-Grained Soils

**Table 5.2** Unified Soil Classification System (Based on Material Passing 76.2-mm Sieve)

Criteria for assigning group symbols				Group symbol
Coarse-grained soils More than 50% of retained on No. 200 sieve	<b>Gravels</b> More than 50% of coarse fraction retained on No. 4 sieve	Clean Gravels	$C_u \geq 4$ and $1 \leq C_c \leq 3^c$	GW
		Less than 5% fines <sup>a</sup>	$C_u < 4$ and/or $C_c < 1$ or $C_c > 3^c$	GP
		Gravels with Fines	$PI < 4$ or plots below "A" line (Figure 5.3)	GM
		More than 12% fines <sup>a,d</sup>	$PI > 7$ and plots on or above "A" line (Figure 5.3)	GC
	<b>Sands</b> 50% or more of coarse fraction passes No. 4 sieve	Clean Sands	$C_u \geq 6$ and $1 \leq C_c \leq 3^c$	SW
		Less than 5% fines <sup>b</sup>	$C_u < 6$ and/or $C_c < 1$ or $C_c > 3^c$	SP
		Sands with Fines	$PI < 4$ or plots below "A" line (Figure 5.3)	SM
		More than 12% fines <sup>b,d</sup>	$PI > 7$ and plots on or above "A" line (Figure 5.3)	SC
Fine-grained soils 50% or more passes No. 200 sieve	<b>Silts and clays</b> Liquid limit less than 50	Inorganic	$PI > 7$ and plots on or above "A" line (Figure 5.3) <sup>e</sup> $PI < 4$ or plots below "A" line (Figure 5.3) <sup>f</sup>	CL ML
		Organic	$\frac{\text{Liquid limit—oven dried}}{\text{Liquid limit—not dried}} < 0.75$ ; see Figure 5.3; OL zone	OL
	<b>Silts and clays</b> Liquid limit 50 or more	Inorganic	$PI$ plots on or above "A" line (Figure 5.3)	CH
			$PI$ plots below "A" line (Figure 5.3)	MH
		Organic	$\frac{\text{Liquid limit—oven dried}}{\text{Liquid limit—not dried}} < 0.75$ ; see Figure 5.3; OH zone	OH
Highly organic soils	Primarily organic matter, dark in color, and organic odor			Pt

**No 200 Sieve  
(Grain-size 0.075mm)**

<sup>a</sup>Gravels with 5 to 12% fine require dual symbols: GW-GM, GW-GC, GP-GM, GP-GC.

<sup>b</sup>Sands with 5 to 12% fines require dual symbols: SW-SM, SW-SC, SP-SM, SP-SC.

$$^c C_u = \frac{D_{60}}{D_{10}}; \quad C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}}$$

<sup>d</sup>If  $4 \leq PI \leq 7$  and plots in the hatched area in Figure 5.3, use dual symbol GC-GM or SC-SM.

<sup>e</sup>If  $4 \leq PI \leq 7$  and plots in the hatched area in Figure 5.3, use dual symbol CL-ML.

# GENERAL SOIL CHARACTERISTICS AND ITS CLASSIFICATION

	<b>Kerikil (gravel), pasir (sand)</b>	<b>Lanau (silt)</b>	<b>Lempung (clay)</b>
Ukuran butir (grain size)	Kasar (coarse), terlihat oleh mata	Fine (halus), tidak terlihat oleh mata	Individual grain
Karakteristik	Non-kohefif, non-plastic, granular	Non-kohefif, non-plastic, granular	Kohefif, plastic
Efek air	Relatif tidak penting, kecuali untuk beban dinamik	Penting	Sangat penting
Efek distribusi ukuran butiran	Penting	Relatif tidak penting	Relatif tidak penting

# **Total and Effective Stress Approach for Solution of Stability Problems in Geotechnical Engineering**

Untuk solusi praktis dalam masalah stabilitas geoteknik, dilakukan 2 pendekatan analisis, yaitu:

- *Total Stress Analysis* (Undrained Condition)
- *Effective Stress Analysis* (Drained Condition)

$$\tau_{ff} = (\sigma_{ff} - u) \tan \phi' + c' \quad ; \quad u = u_0 + \Delta u$$

**Undrained**

$\Delta u$

(End of Construction)

time

**Drained**

$\Delta u = 0$

(Long-Term)

Dengan kriteria keruntuhan Mohr-Coulomb kita dapat menghitung tegangan-tegangan pada bidang runtuh pada saat keruntuhan terjadi dan mengevaluasi Factor of Safety (FoS):

$$FoS = \tau_{ff} \text{ (yang ada)} / \tau_f \text{ (yang bekerja)}$$

## **Effective Stress Analysis**

- **Consistently:** use of effective stress and effective strength and modulus parameters.
- **Consequently:** Need to calculate  $u = u_0 + \Delta u$  for any condition from undrained to drained conditions (It is also applicable for short-term (end of construction) stability analysis as long as  $\Delta u$  is available).
- **Practical for Long-Term stability condition,** since  $u$  is easy to evaluate and  $\Delta u = 0$ .

## **Total Stress Analysis**

- **Consistently:** use total stress and total strength and modulus parameters.
- **Advantage:** no need to compute pore water pressure  $u$
- **Very practical Short-Term (end of construction) stability condition.**

# Undrained Analysis With Effective Parameters

It is possible to specify undrained behavior in an effective stress analysis using effective model parameters. This is achieved by identifying the type of material behavior (or material type) of a soil layer as undrained.

The presence of pore water pressure in a soil body, usually caused by water, contributes to the total stress level. According to Terzaghi's principle, total stress  $\underline{\sigma}$  can be divided into effective stress  $\underline{\sigma}'$  and pore pressures  $\underline{\sigma}_w$ :

$$\sigma_{xx} = \sigma'_{xx} + \sigma_w$$

$$\sigma_{yy} = \sigma'_{yy} + \sigma_w$$

$$\sigma_{zz} = \sigma'_{zz} + \sigma_w$$

$$\sigma_{xy} = \sigma'_{xy}$$

$$\sigma_w = u = u_0 + \Delta u$$

$u$  = pore water pressure (pwp)

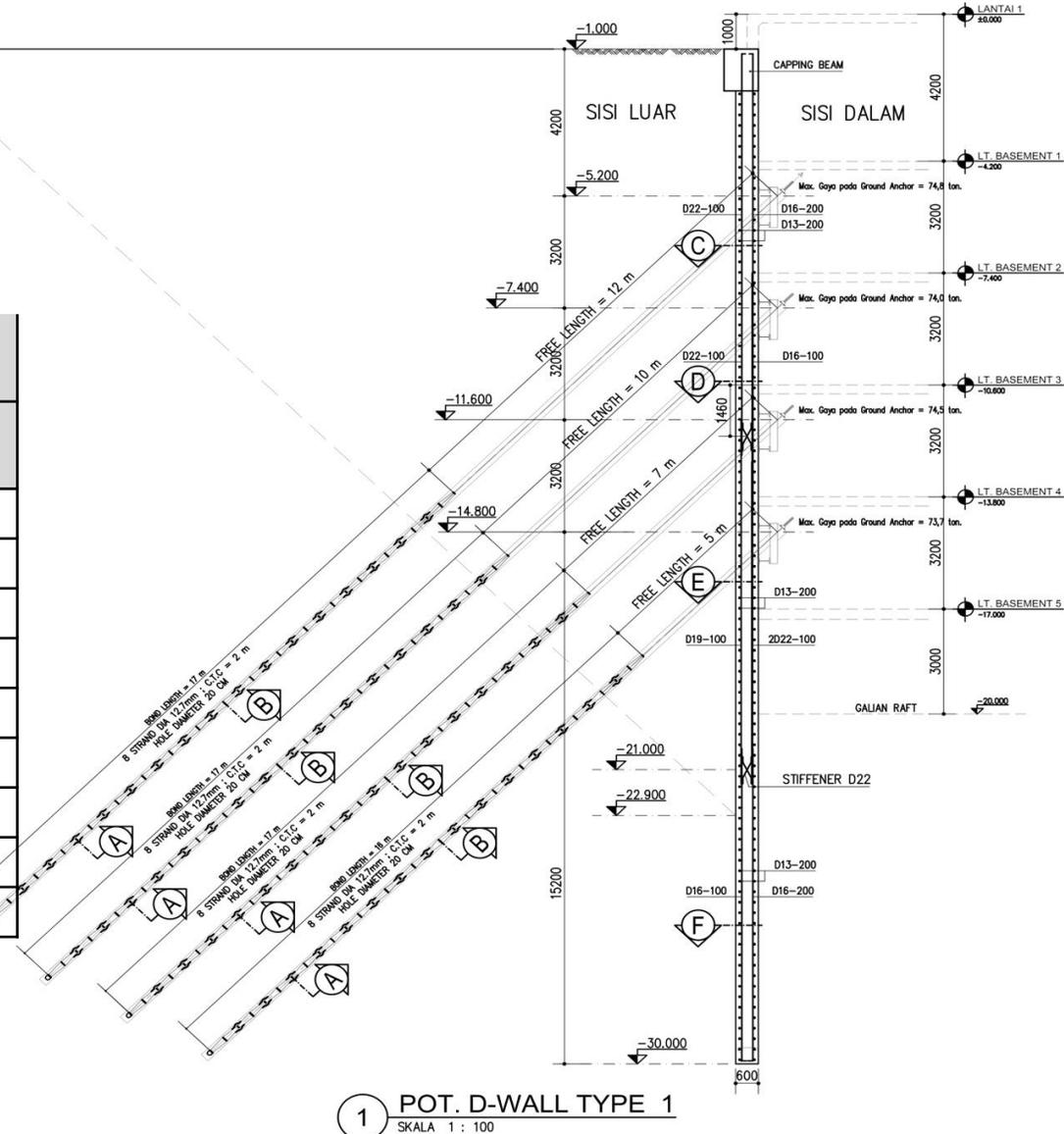
$u_0$  = initial pwp (hydrostatic or seepage)

$\Delta u$  = excess pwp due to change in stress

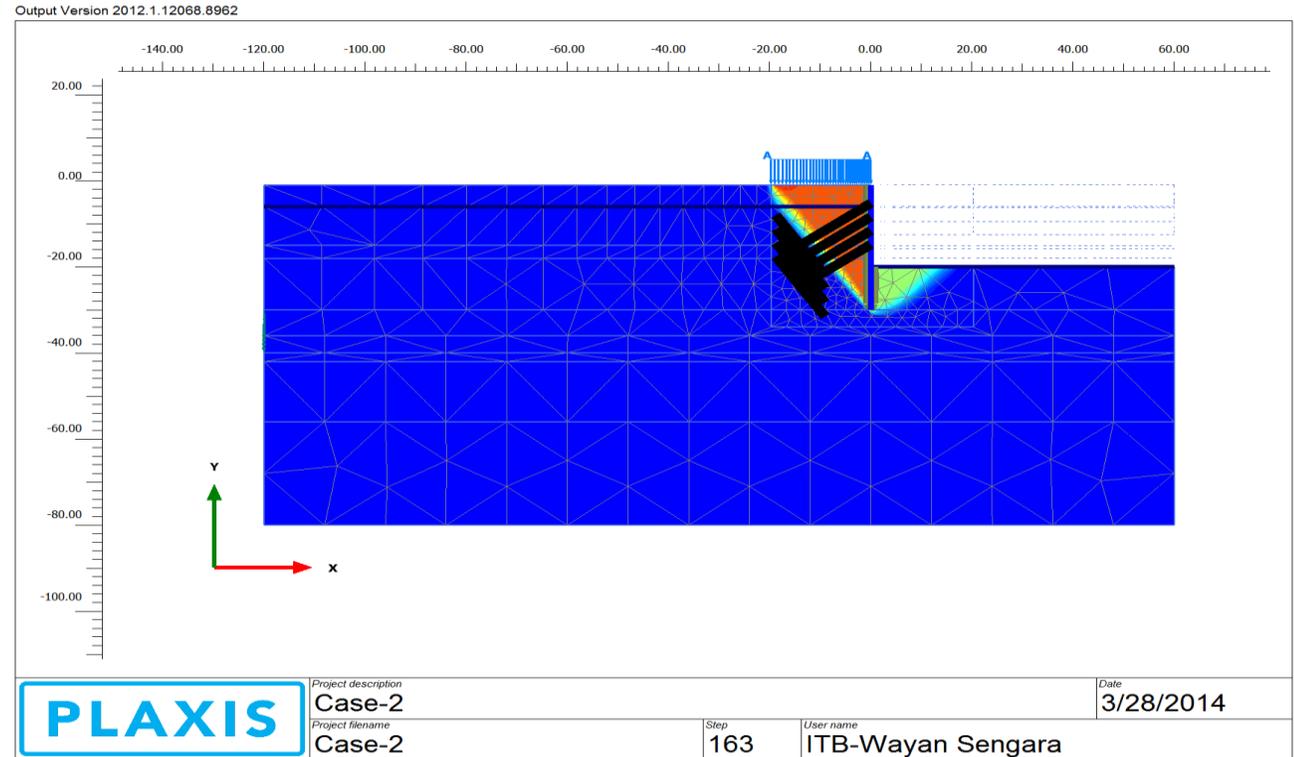
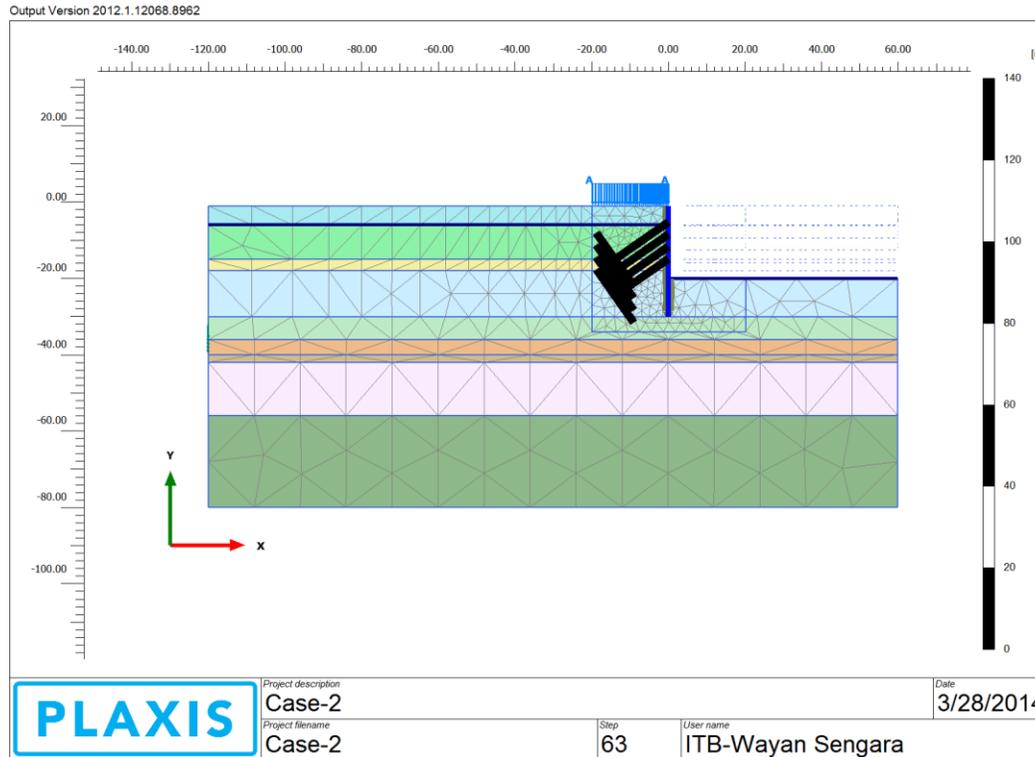
# Typical Needs of Shear Strength Parameters in a Case Modeling of Deep Excavation

Soil Parameters

Depth (m)	N-SPT average	Soil Classification	Undrained Parameter		Drained Parameter		
			$c_u$ (kPa)	$E_u$ (kPa)	$c'$ (kPa)	$\phi'$ (°)	$E'$ (kPa)
0	5	Silty Clay	26	3900	5	18	2613
5	14	Sand, Very Dense	-	-	20	30	44625
14	17	Silty CLAY	200	70000	10	30	46900
17	29	Silty CLAY	200	70000	20	30	46900
29	35	Cemented Sand	-	-	20	30	45000
35	39	Silty Clay	96	24000	-	-	-
39	41	Sand, Medium Dense	-	-	5	31	16500
41	55	Silty CLAY	200	70000	-	-	-
55	60	Cemented Sand	-	-	20	40	40500



# Case Finite Element Modeling of Deep Excavation



Wall Lateral Deformation (cm)	Bending Moment (tm/m)	Max. Force on Anchor		Safety factor, SF	
		Plane Strain (t/m')	Spacing 2 m (t)	Undrained	Drained
5.7 – 11.9	37.9 - 68.1	37.6	75.2	1.56	1.29
		38.1	76.2		
		38.6	77.2		
		38.4	76.8		



## **Need Understanding of:**

- Stress at a Point
- Stress – Strain Characteristics
- Failure Criterion
- Stress Path

(Reference: An Introduction to Geotechnical Engineering by Robert D. Holtz and William D. Kovacs)

# INTRODUCTION

- The shear strength of soils is a most important aspect of geotechnical engineering. The bearing capacity of shallow or deep foundations, slope stability, retaining wall design and indirectly, pavement design are all affected by the shear strength of the soil in a slope, behind a retaining wall, or supporting a foundation or pavement. Structures and slopes must be stable and secure against total collapse when subjected to maximum anticipated applied loads. Thus limiting equilibrium methods of analysis are conventionally used for their design, and these methods require determination of the ultimate or limiting shear resistance (shear strength) of the soil.
- The shear strength of a soil defined as the ultimate or maximum shear stress the soil can withstand. We mentioned that sometimes the limiting value of shear stress was based on a maximum allowable strain or deformation. Very often, this allowable deformation actually controls the design of a structure because with the large safety factors we use, the actual shear stresses in the soil produced by the applied loads are much less than the stresses causing collapse or failure.

# FUNDAMENTALS CONCEPTS: STRESS AND FAILURE

The analysis of stability problems in Geotechnical Engineering, such as bearing capacity of foundations, retaining structures, and slope stability, requires a knowledge of the shear strength of the soils involved.

These analysis are based on conditions of limiting equilibrium which requires a comparison of the state of stress in the soil with the failure state of stress which is defined as soil strength along an assumed failure plane. Therefore, the first step is to understand soil stresses.

# STRESS AT A POINT

- The concept of stress at a point in a soil is really fictitious. The point of application of a force within a soil mass could be on a particle or in a void. Clearly, a void cannot support any force, but if the force were applied to a particle, the stress could be extremely large. Thus when we speak about stress in the context of soil materials we are really speaking about a force per unit area, in which the area under consideration is the gross cross-sectional or engineering area. This area contains both grain-to-grain contacts as well as voids.
- Consider a soil mass that is acted upon by a set of forces  $F_1, F_2, \dots, F_n$ , as shown in Figure below. For the time being, let's assume that these forces act in a two-dimensional plane. We could resolve these forces into components on a small element at any point within the soil mass, such as point  $O$  in that Figure.

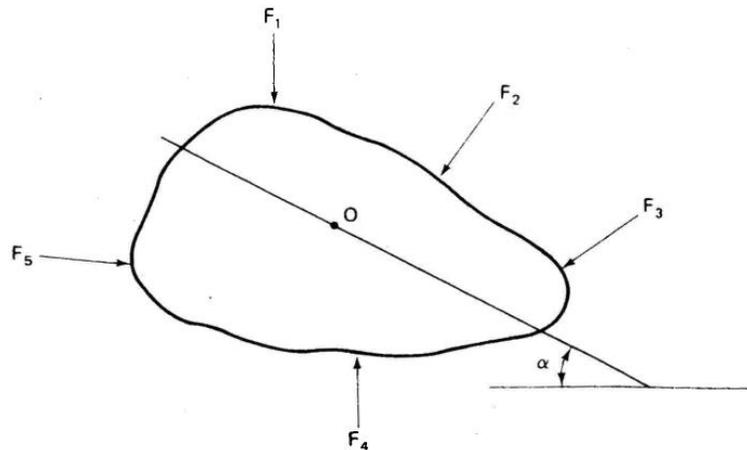


Fig. 10.1 A soil mass acted upon by several forces.

# STRESS AT A POINT

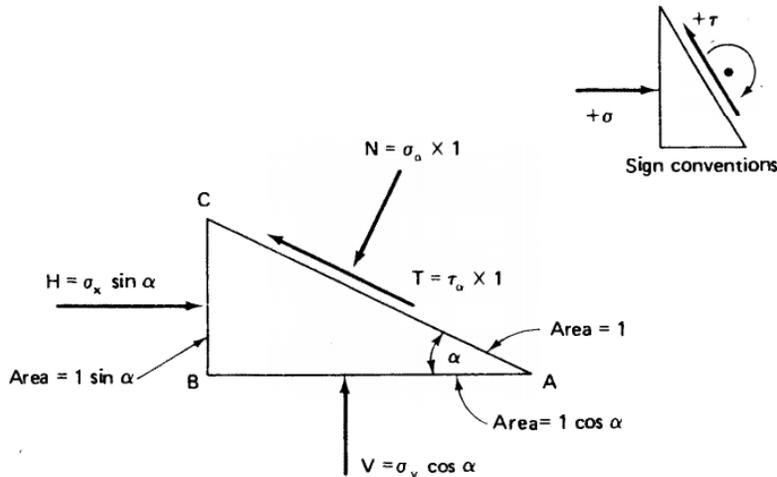


Fig. 10.2 Resolution of the forces of Fig. 10.1 into components on a small element at point O. Sign conventions are shown in the small inset figure.

- To begin, let's assume that the distance AC along the inclined plane in Figure beside has unit length, and that the figure has a unit depth perpendicular to the plane of the paper. Thus the vertical plane BC has the dimension of  $1 \sin \alpha$ , and the horizontal dimension AB has a dimension equal to  $1 \cos \alpha$ . At equilibrium, the sum of the forces in any direction must be zero. So summing in the horizontal and vertical directions, we obtain.

$$\Sigma F_h = H - T \cos \alpha - N \sin \alpha = 0$$

$$\Sigma F_v = V + T \sin \alpha - N \cos \alpha = 0$$

- Dividing the forces in Equation above by the areas upon which they act, we obtain the normal and shear stresses. (We shall denote the horizontal normal stress by  $\sigma_x$  and the vertical normal stress by  $\sigma_y$ ; the stresses on the  $\alpha$ -plane are normal stress  $\sigma_\alpha$  and the shear stress  $\tau_\alpha$ )

$$\sigma_x \sin \alpha - \tau_\alpha \cos \alpha - \sigma_\alpha \sin \alpha = 0$$

$$\sigma_y \cos \alpha + \tau_\alpha \sin \alpha - \sigma_\alpha \cos \alpha = 0$$

- Solving equation above simultaneously for  $\sigma_\alpha$  and  $\tau_\alpha$ , we obtain:

$$\sigma_\alpha = \sigma_x \sin^2 \alpha + \sigma_y \cos^2 \alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha$$

$$\tau_\alpha = (\sigma_x - \sigma_y) \sin \alpha \cos \alpha = \frac{\sigma_x - \sigma_y}{2} \sin 2\alpha$$

# STRESS AT A POINT

- Since the vertical and horizontal planes have no shearing stresses acting on them, they are by definition **principal planes**. Thus the stresses  $\sigma_x$  and  $\sigma_y$  are really **principal stresses**. You may recall from your study of strength of materials that principal stresses act on planes where  $\tau = 0$ . the stress with the largest magnitude is called the *major principal stress*, and denoted by  $\sigma_1$ . the smallest principal stress is called the *minor principal stress*,  $\sigma_3$ , and the stress in the third dimension is the *intermediate principal stress*,  $\sigma_2$ . In figure below,  $\sigma_2$  is neglected since our derivation was for two-dimensional plane stress conditions.

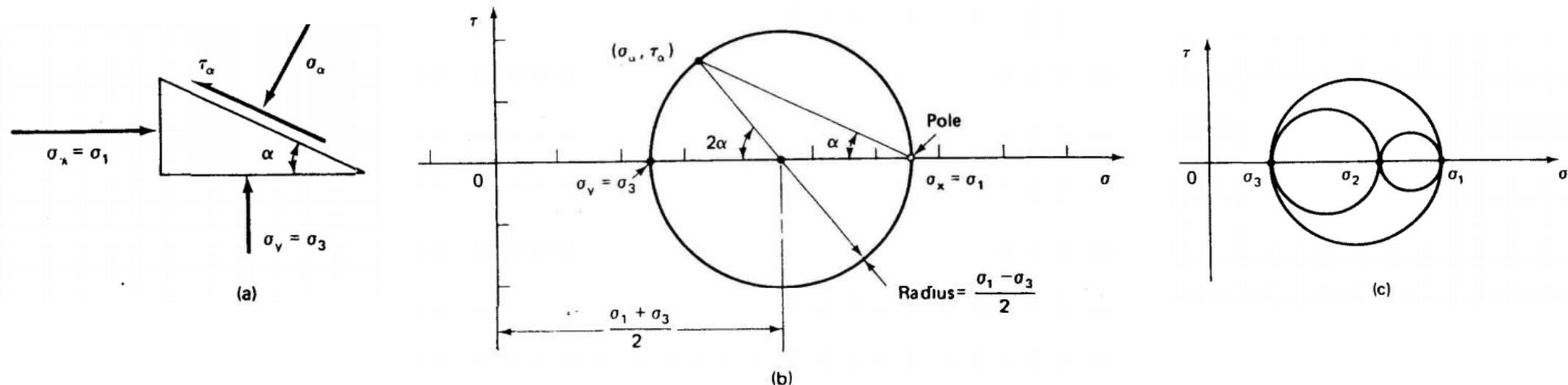


Fig. 10.3 The Mohr circle of stress: (a) element at equilibrium; (b) the Mohr circle; (c) Mohr circles including  $\sigma_2$ .

# STRESS – STRAIN CHARACTERISTICS OF SOILS

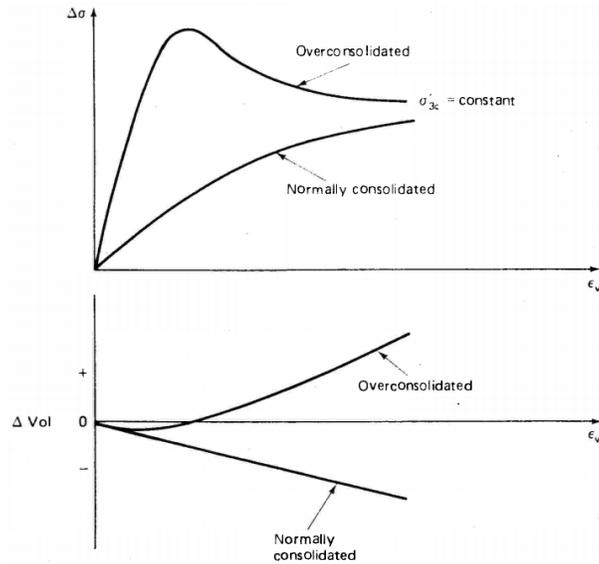
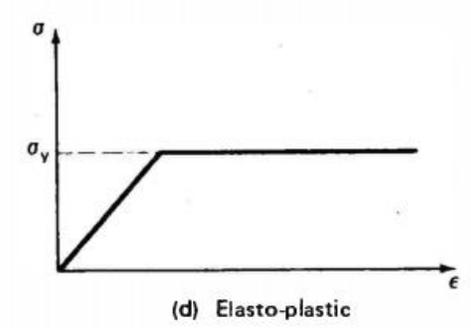
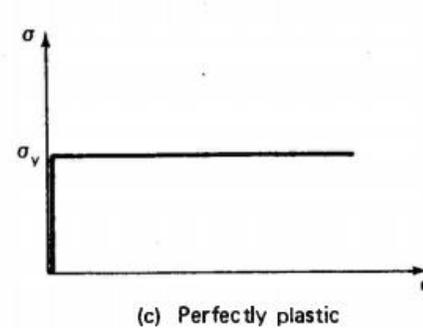


Fig. 11.24 Typical stress-strain and volume change versus strain curves for CD axial compression tests at the same effective confining stress.



- **Soils have a highly non-linear stress-strain-time behavior.**
- Soil could be modeled as **perfectly plastic** materials, sometimes called **rigid-plastic**, can be treated relatively easily mathematically
- A more realistic stress-strain relationship is **elasto-plastic**. The material is linearly elastic up to the yield point  $\sigma_y$ ; then it becomes perfectly plastic. Note that both perfectly plastic and elasto-plastic materials continue to strain even without any additional stress applied.

# STRESS – STRAIN CHARACTERISTICS

- At what point on the stress-strain curve do we have **failure**? We could call the yield point “failure” if we wanted to. In some situations, if a material is stressed to its yield point, the strains or deflections are so large that for all practical purposes the material has failed. This means that the material cannot satisfactorily continue to carry the applied loads.
- The stress at “failure” is often very arbitrary, especially for nonlinear materials. With brittle type materials, however, there is no question when failure occurs – it’s obvious. Even with work-softening materials, the peak of the curve or the maximum stress is usually defined as failure. On the other hand, with some plastic materials it may not be obvious. Where would you define failure if you had a work-hardening stress-strain curve? With materials such as these, we usually define failure at some arbitrary percent strain, for example **15 to 20%** or at a strain or deformation at which the function of the structure might be impaired.
- **Failure:** maximum or yield stress or the stress at some strain.

# FAILURE CRITERION

- Mohr is the same Otto Mohr of Mohr circle fame. Coulomb you know from coulombic friction, electrostatic and repulsion, among other things. Around the turn of this century, Mohr (1900) hypothesized a criterion of failure for real materials in which he stated that materials fail when the shear stress on the failure plane at failure reaches some unique function of the normal stress on that plane, or:
- $\tau_{ff} = f(\sigma_{ff})$
- Where  $\tau$  is the shear stress and  $\sigma$  is the normal stress. The first subscript  $f$  refers to the plane on which the stress acts (in this case the failure plane) and the second  $f$  means “at failure”.
- $\tau_{ff}$  is called the shear strength of the material and the relationship is shown in Figure below.

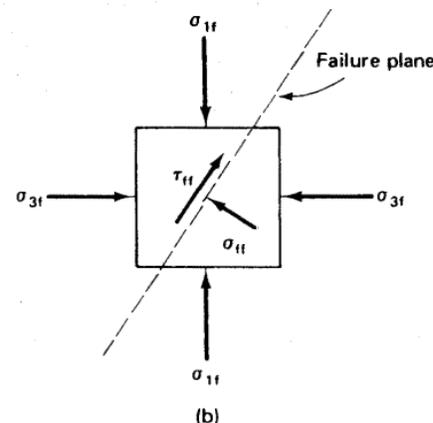
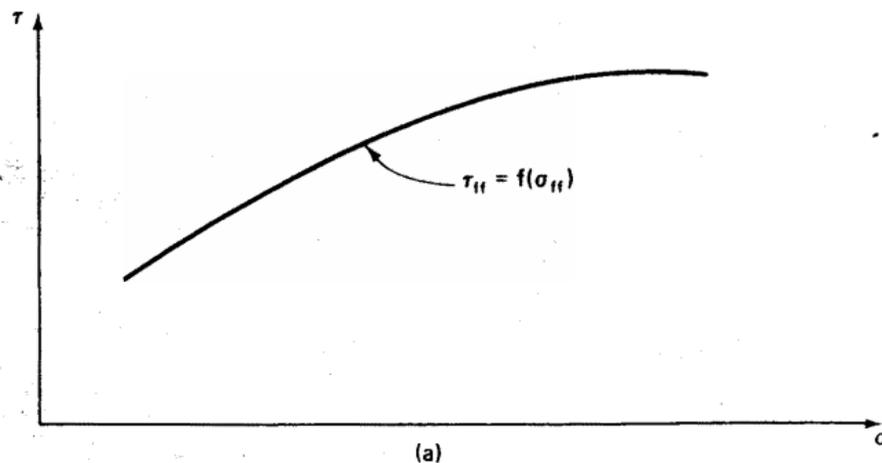


Fig. 10.5 (a) Mohr failure criterion; (b) element at failure, showing the principal stresses and the stresses on the failure plane.

# FAILURE CRITERION

- Note that any Mohr circle lying below the Mohr failure envelope such as circle A in figure below represents a stable condition. Failure occurs only when the combination of shear and normal stress is such that the Mohr circle is tangent to the Mohr failure envelope. Note also that circles lying above the Mohr failure envelope (such as circle B) cannot exist.
- The material would fail before reaching these states of stress. If this envelope is unique for a given material, then the point of tangency of the Mohr failure envelope gives the stress conditions on the failure plane at failure. Using the pole method, we can therefore determine the angle of the failure plane from the point of tangency of the Mohr circle and the Mohr failure envelope.

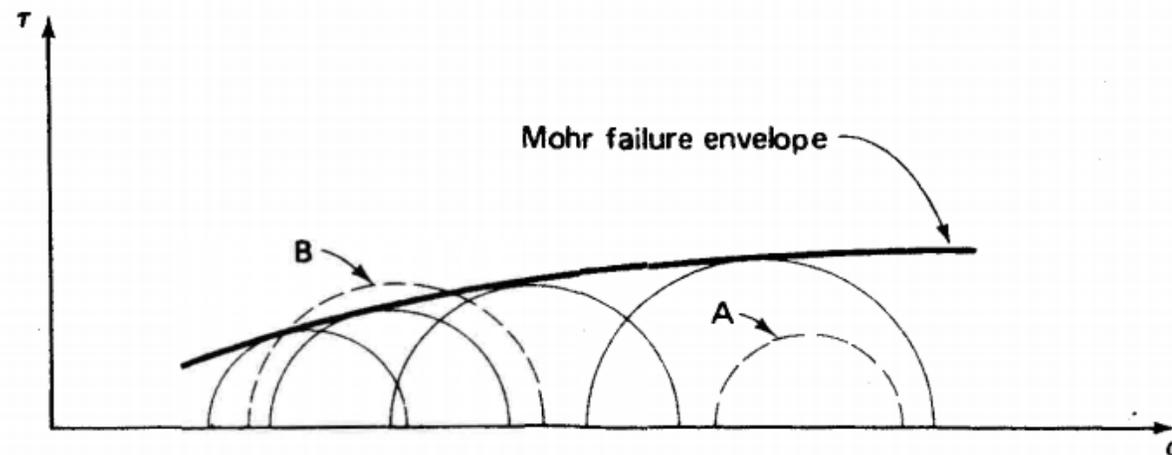


Fig. 10.6 The Mohr circles at failure define the Mohr failure envelope.

# FAILURE CRITERION

The Mohr – Coulomb criterion is most widely used to define failure in soils. According to this criterion the shear strength can be expressed consistently in terms of effective stress as

$$s = c' + \sigma' \tan \phi' = c' + (\sigma - u) \tan \phi'$$

Where  $c'$  and  $\phi'$  are the effective strength parameters: cohesion intercept and angle of internal friction, respectively:

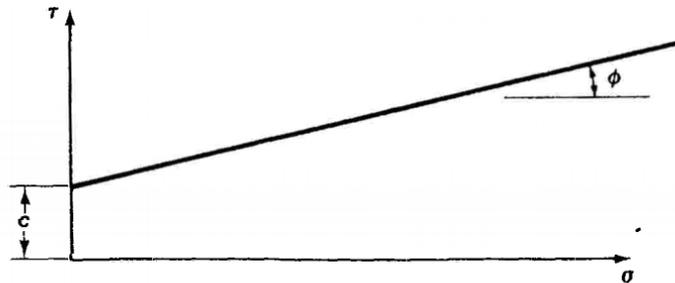
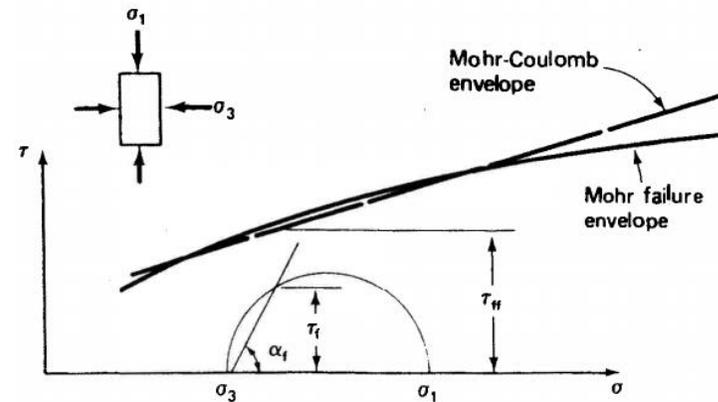


Fig. 10.8 The Coulomb strength equation presented graphically.



# FAILURE CRITERION

- The hypothesis, that the point of tangency defines the angle of the failure plane in the element or test specimen, is the *Mohr failure hypothesis*. You should distinguish this hypothesis from the Mohr failure theory.
- The Mohr failure hypothesis is illustrated in Figure (a) below for the element at failure shown in Figure (b). Stated another way: the Mohr failure hypothesis states that the point of tangency of the Mohr failure envelope with the Mohr circle at failure determines the inclination of the failure plane.

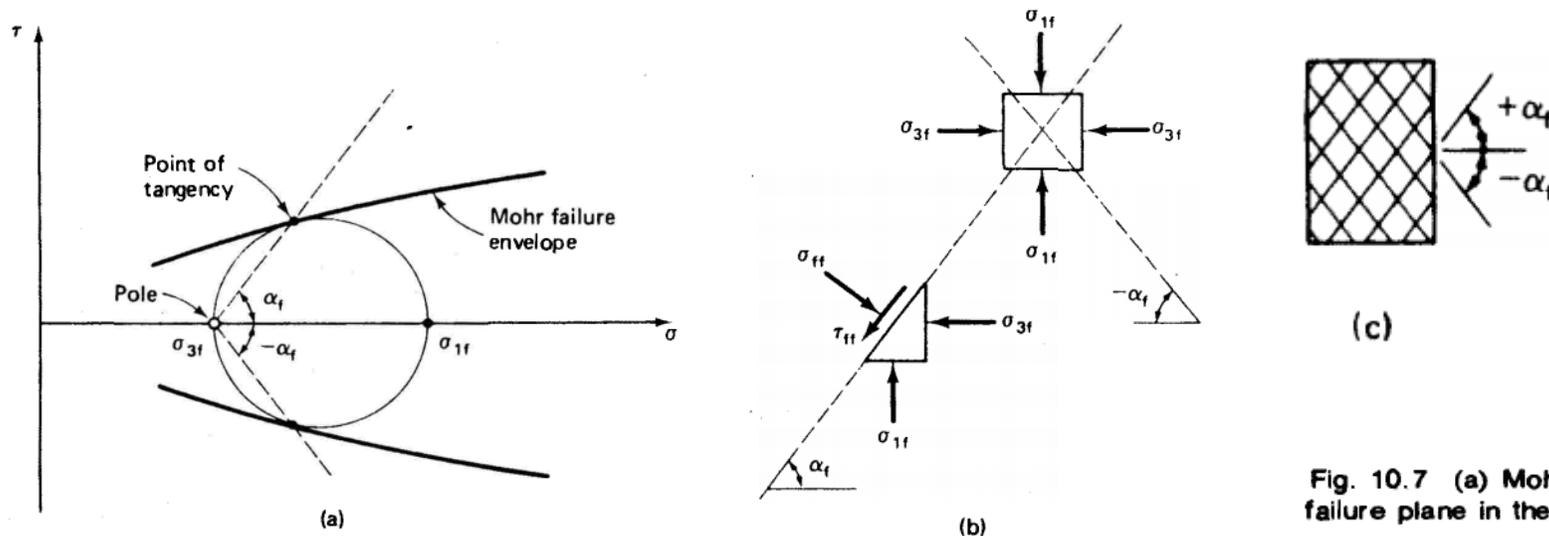


Fig. 10.7 (a) Mohr failure hypothesis for determining the angle of the failure plane in the (b) element; (c) conjugate failure planes.

# FAILURE CRITERION

$$s = c' + \sigma' \tan \phi' = c' + (\sigma - u) \tan \phi'$$

As indicated by the equation above, the failure in soils is caused by a critical **combination of both shear and normal stresses**. Failure is essentially by shear, but critical shear stress is governed by the normal stress acting on the potential surface of failure. The line which plots this critical combination in a  $\tau - \sigma$  plot is known as “failure envelope” – Mohr Circle gives all possible combinations of shear and normal stresses. Failure occurs on the plane represented by the intersection of the circle with the envelope.

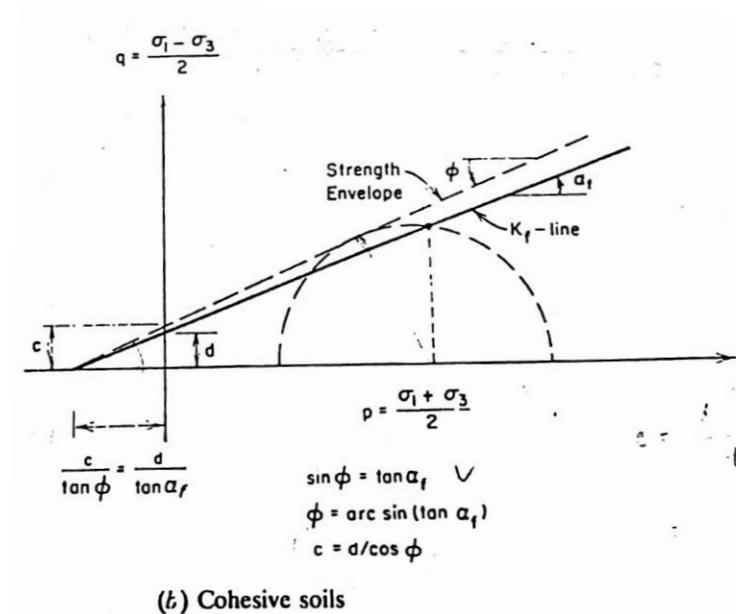
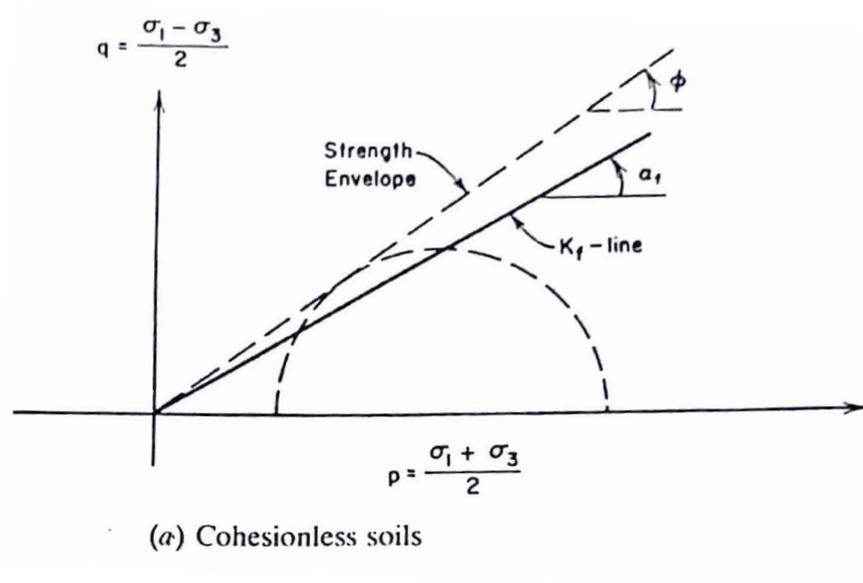
The envelope will not necessarily be a straight line, and thus  $c'$  and  $\phi'$  would represent a straight line approximation to the actual envelope over the stress range of interest. Furthermore, the envelope will not necessarily have a cohesion intercept, i.e.  $c' = 0$ . In fact, the envelope is not an unique quantity for a given soil, but is a function of several variables, the most important being stress history.

# FAILURE CRITERION

When employing stress paths, it is more convenient to use a modified failure envelope based on a plot of  $q_f$  vs  $p'_f$ . Thus:

$$q_f = a' + p'_f \tan \alpha'$$

Where:  $c' = a' / \cos \phi'$  and  $\sin \phi' = \tan \alpha'$



# FAILURE CRITERION

- The use of effective strength parameters requires that the pore-pressure ( $u = u_o + \Delta u$ ) is known so that  $\sigma'$  may be evaluated.
- Under fully drained, long-term condition, the pore-pressure change due to applied loads ( $\Delta u$ ) is zero, and pore pressure due to ground water flow ( $u_o$ ) can usually be evaluated without serious difficulty. Hence, analysis with the effective stress description of shear strength is most useful.
- For partially drained & undrained conditions, the evaluation of  $\Delta u$  is often difficult. In some cases, a total stress description of shear strength may be used as  $s = s_u$
- One important case is the undrained loading of saturated cohesive soils. In this case, the undrained strength ( $s_u$ ) can be used, where  $s_u = c_u$  and  $\varphi' = 0$ . The shear strength usually changes as drainage occurs. If the change results in a higher strength, the short-term, undrained stability is critical and stability can be expected to improve with time. On the other hand, if drainage produces a decrease in strength, the undrained shear strength can be used only for short-term or temporary situations.

# STRESS PATHS

The Mohr diagram can be useful for representing a series of stress states by drawing several Mohr circles showing progressive changes in the state of stress during construction or load application. It is convenient, for such cases, to replace a Mohr circle by a single point, for example, the point of maximum shear stress (top of the circle) having the coordinates.

**A stress path** is defined as the locus of points on the Mohr diagram whose coordinates represent the maximum shear stress and the associated mean principal (or normal) stress plotted for the entire stress history of a soil element.

$$p = \frac{\sigma_1 + \sigma_3}{2}$$

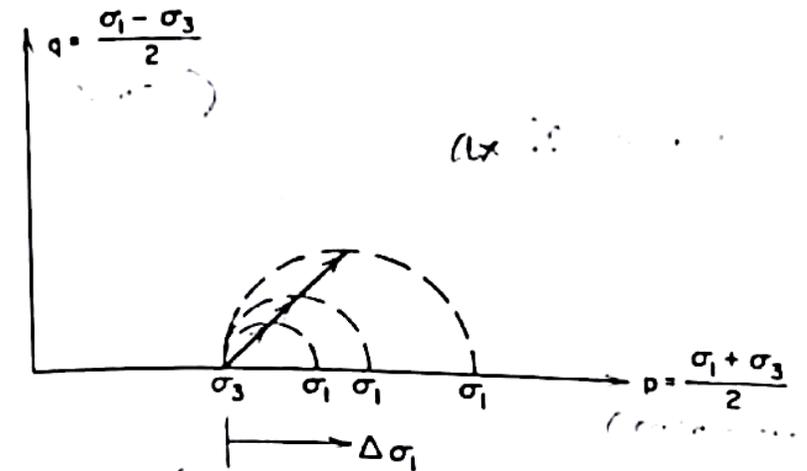
$$q = \frac{\sigma_1 - \sigma_3}{2}$$

For field stress condition:

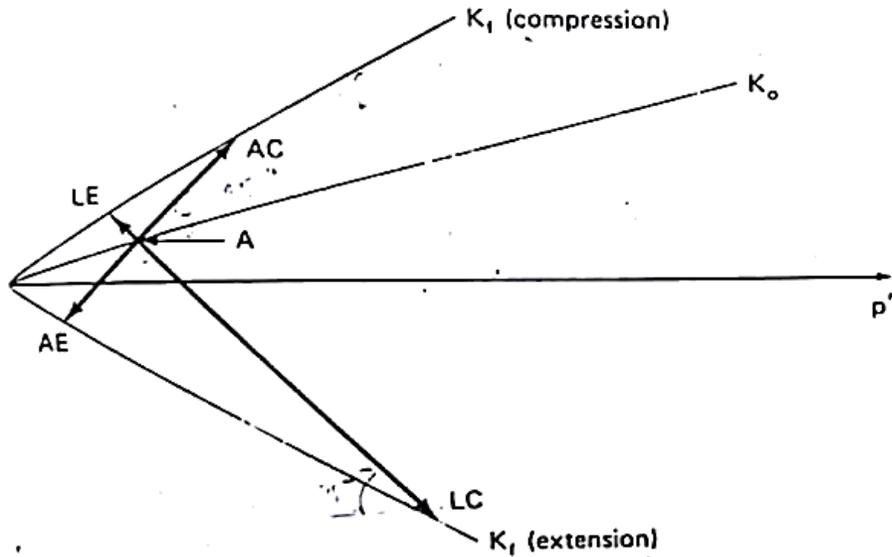
$q$  is positive when  $\sigma_v > \sigma_h$

$$p = \frac{\sigma_v + \sigma_h}{2}$$

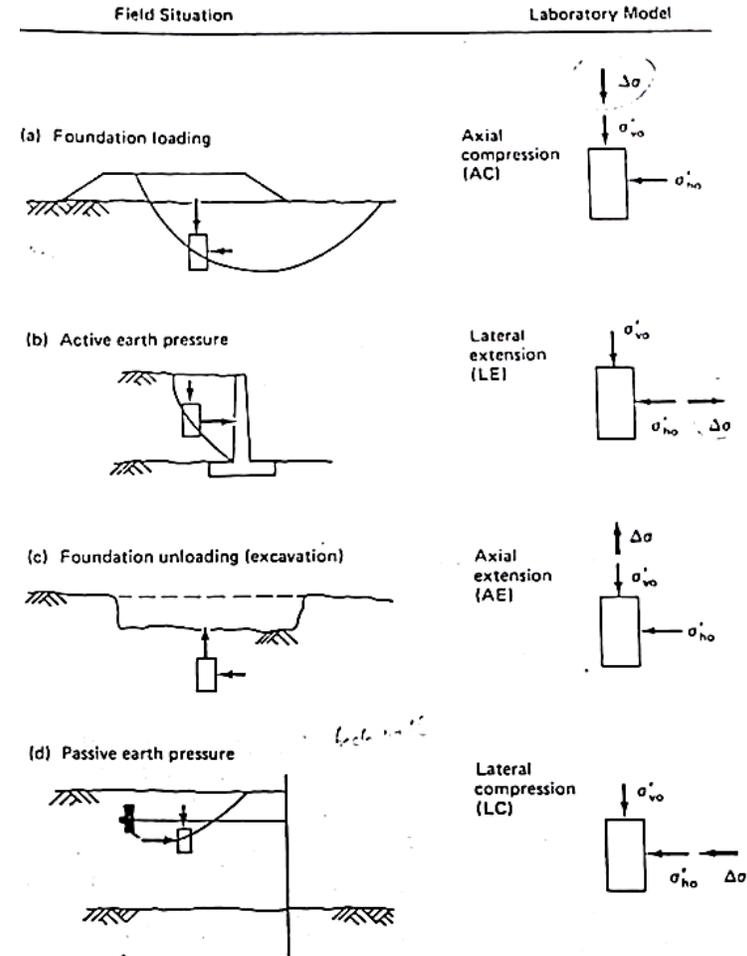
$$q = \frac{\sigma_v - \sigma_h}{2}$$



# STRESS PATHS

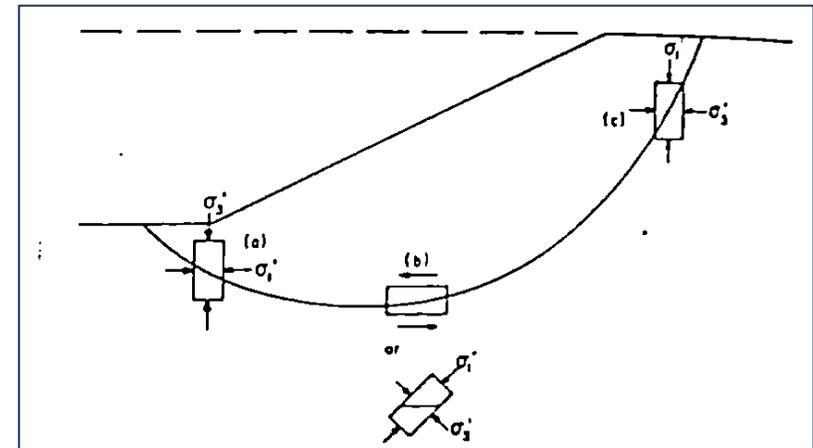
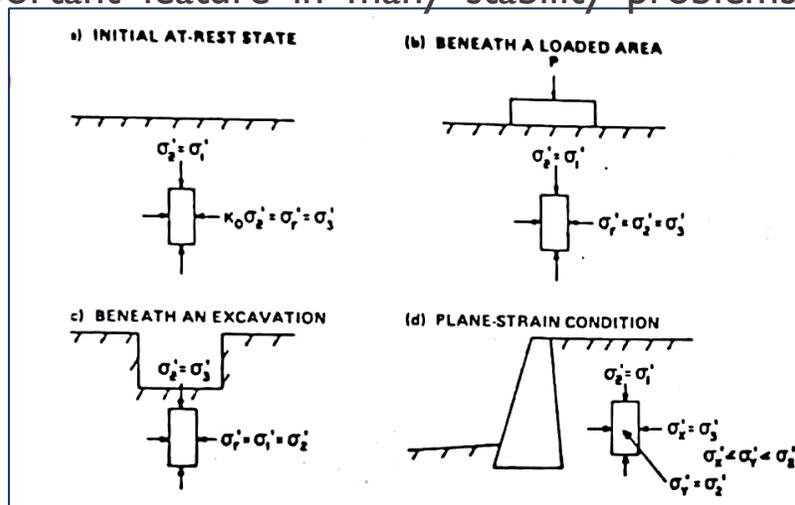


Symbol	Geotechnical Engineering example
Axial Compression	Foundation loading – increase $\sigma_v$ , $\sigma_h$ constant
Lateral Extension	Active earth pressure – decrease $\sigma_h$ , $\sigma_v$ constant
Axial Extension	Unloading (excavation) – decrease $\sigma_v$ , $\sigma_h$ constant
Lateral Compression	Passive earth pressure – increase $\sigma_h$ , $\sigma_v$ constant



# COMMON STATES OF STRESS

- In the initial state,  $\sigma_z'$  is the overburden pressure,  $\sigma_r' = K_0 \sigma_z'$  is the lateral pressure, and  $K_0$  is the coefficient of earth pressure at-rest. In the stress state beneath the center of a circular loaded area, the vertical stress ( $\sigma_z' = \sigma_{z'o}' + \Delta\sigma_z'$ ) is the major principal stress and the radial stress ( $\sigma_r'$ ) is the minor principal stress (compression-loading). In the stress state below the center of a circular excavation, the vertical stress is the minor principal stress and the radial stress is the major principal stress (extension – unloading). For the circular load, the intermediate principal stress ( $\sigma_2'$ ) is equal to the minor principal stress ( $\sigma_3'$ ) ; for the excavation, it is equal to the major principal stress ( $\sigma_1'$ ). Slopes and retaining structures can be approximated by the plane –strain condition in which the intermediate principal strain ( $\epsilon_2$ ) is zero. The active condition corresponds to compression – unloading and passive condition corresponds to extension – loading for retaining structures.
- Another important feature in many stability problems is the rotation of the principal axis axes during loading or excavation.



# APPLICATIONS OF STRESS PATHS TO ENGINEERING PRACTICE

- Consider the case of foundation loading, for example embankment constructed on a soft clay foundation. Assume that the clay is very nearly 100% saturated and is normally consolidated.

=> This case may be modeled by axial compression stress conditions.

- The loading is assumed as plane-strain for a long embankment, but we use the common triaxial test, for illustrative purposes.

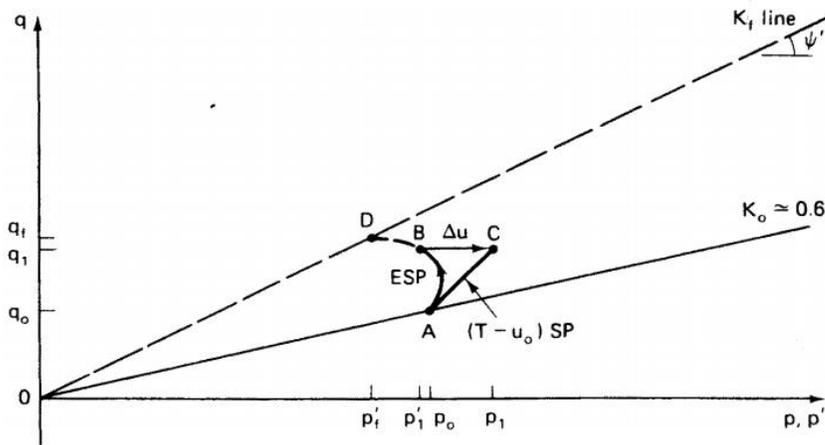


Fig. 11.82 Stress paths for (a) foundation loading

- For this NC clay, the  $K_0$  is less than 1 (about 0.6), so that the initial stress conditions in the ground are plotted as point A.
- In a foundation loading, the horizontal stresses probably increase slightly, but for this case we will assume that they are essentially constant.
- The total stresses represented by point C are applied at the end of construction.
- The induced pore pressure are positive and so we will have the typical ESP hooking off to the left.
- If the loading continued to the level of  $q_f$ , the ESP would have intersected the  $K_f$  line and failure will occurred.

# APPLICATIONS OF STRESS PATHS TO ENGINEERING PRACTICE

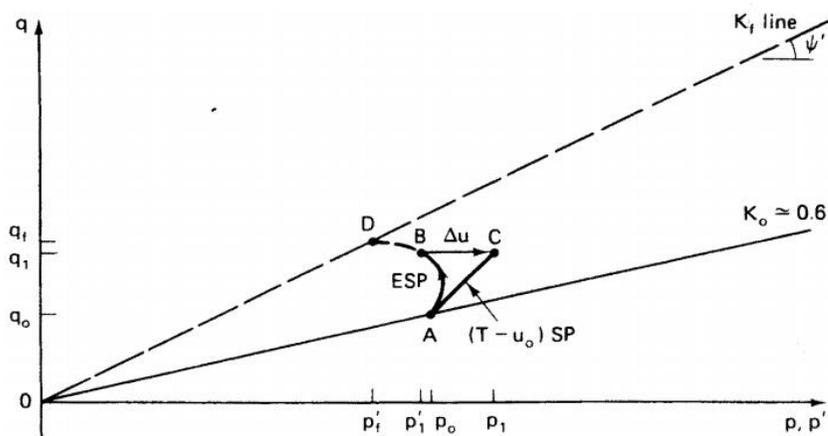


Fig. 11.82 Stress paths for (a) foundation loading

- Let's talk at the point B on the ESP at the end of construction, the most critical design condition for foundation loadings on NC clays.
- Look at what happens after we reach point B. The applied loadings is constant thereafter (assuming no additional construction occurs), the clay starts to consolidate, and the excess pore water pressure that was caused by the load dissipates.
- This excess pore pressure is represented by the distance BC. Ultimately, at  $u = 100\%$ , all the excess pore pressure will be dissipated and our element will be at point C in equilibrium under the embankment load.
- Since there is no excess pore water pressure remaining in the element, the total stresses will equal the effective stresses at point C.
- Now you can see why point B at the end of construction was the most critical for this case. Point B was the closest point to the failure line  $K_f$ .
- The engineering lesson here is that if you make it through the end of construction period for this type of loading, then conditions become *safer* with time.

# STRENGTH CHARACTERISTICS AND MEASUREMENT

- Shear strength is measured both through field and laboratory tests. Laboratory tests are made on representative soil samples and must be done in a way that simulates the conditions that will exist in the field as closely as possible, in particular the drainage and stress conditions.
- The shear strength of granular soils (clean sands and gravels) can generally be made on disturbed samples that are reconstituted in the laboratory to field densities. Field tests through **SPT, CPT, and PMT**.
- However, disturbance significantly affects the physical properties of cohesive soils (plastic silts and clays, organic soils) even if the field density is maintained, laboratory test on cohesive soils must therefore be made on undisturbed samples if the strength of a natural soil deposit is to be determined. Field tests through **SPT, CPT, VST, and PMT**.
- The strength of proposed compacted earth embankments is often required, and for such cases the laboratory samples must be prepared to duplicate the density, water content, and compaction method of the field soil.

# SHEAR STRENGTH OF GRANULAR SOILS

- Clean sands and gravels (fines less than 5-10% by weight) are referred as granular (or sometimes cohesionless) soils and are characterized by high permeability. Therefore, their strength is expressed in terms of effective stresses with  $\Delta u$  equal to zero since  $\Delta u$  would dissipate readily under most quasi-static construction activities (drained condition). The only exception to this is dynamic loading such as earthquake or blasting during which in certain sands  $\Delta u$  may build up faster than it can dissipate. For silty cohesionless soils permeability maybe sufficiently low that  $\Delta u$  may develop during construction.
- In the effective strength computations, only pore pressure due to ground water ( $u_o$ ) must be estimated since  $\Delta u=0$  (drained).  $U_o$  may be positive (due to static or flowing ground water) or negative (due to capillary tension). **There are no cohesive forces (electrical forces) between the grains of granular soils.** However, if confined, such soils offer resistance to shearing proportional to the effective confining pressure. Shear strength can be expressed as

$$s = \sigma' \tan \phi' = (\sigma - u_o) \tan \phi'; \quad c' = 0$$

- Angle of friction has basically two components; one due to inter-particle friction, the other due to interlocking.  $\phi'$  depends on, among other factors, on relative density ( $D_r$ ), grain size distribution, and grain shape (roundness). The value of  $\phi'$  ranges normally from about  $27^\circ$  to  $42^\circ$  or more. The effect of moisture on  $\phi'$  (not on  $s$ ) is small and amounts to no more than  $1^\circ - 2^\circ$ .

# SHEAR STRENGTH TRIAXIAL TESTS:

1. Triaxial Consolidated-Drained (CD) Test
2. Triaxial Consolidated-Undrained (CU) Test
3. Triaxial Unconsolidated-Undrained (UU) Test

- Three general stage on triaxial test are
- Sampling Stage
  - Isotropic Loading Stage (saturation and confining pressure)
  - Shearing Stage

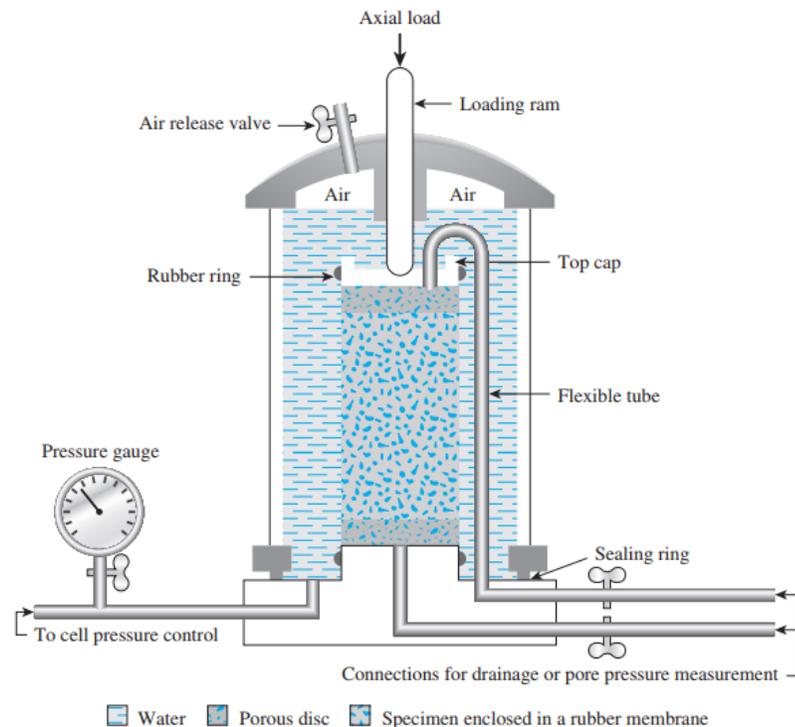
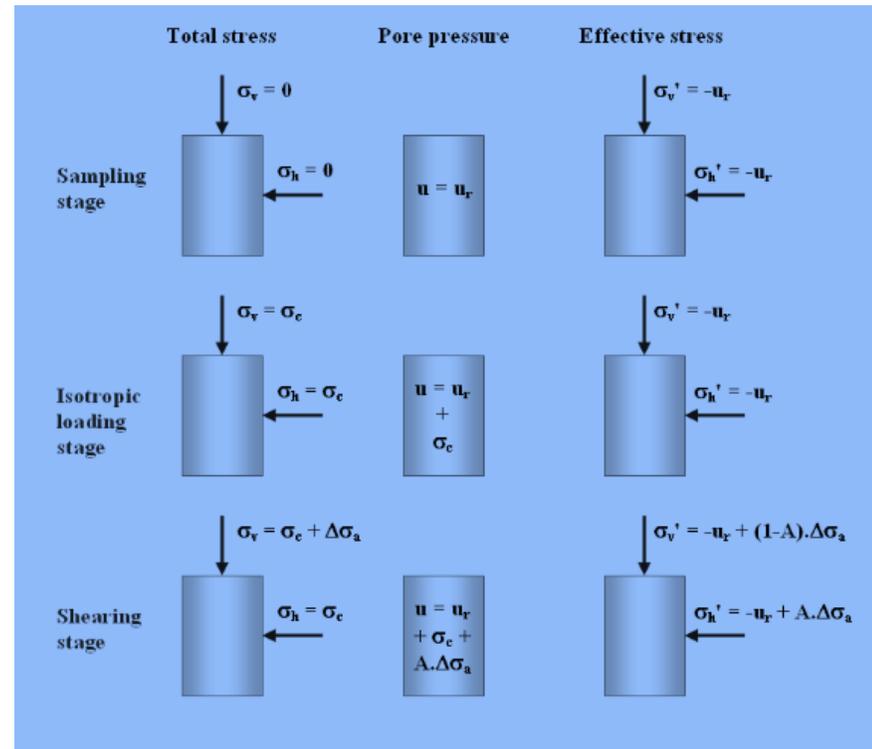


Figure 12.20 Diagram of triaxial test equipment (After Bishop and Bjerrum, 1960. With permission from ASCE.)



# BEHAVIOR OF SATURATED SANDS DURING DRAINED SHEAR

- To illustrate the behavior of sands during shear, let's start by taking two samples of sand, one at a very high void ratio, the "loose" sand, and the other at a very low void ratio, the "dense" sand. We shall run the two **Triaxial Tests** under **Consolidated Drained (CD)** conditions, which means we will allow water to freely enter or leave the sample during shear without interference. If we have a saturated sample, we can easily monitor the amount of water that enters or leaves the sample and equate this to volume change and thus the void ratio change in the sample. Water leaving the sample during shear indicates a volume decrease, and vice versa. In both our tests the confining pressure,  $\sigma_c$ , is held constant and the axial stress is increased until failure occurs. Failure may be defined as:

1. Maximum principal stress difference,  $(\sigma_1 - \sigma_3)_{\max}$
2. Maximum principal effective stress ratio  $(\sigma_1'/\sigma_3')_{\max}$
3.  $\tau = [(\sigma_1 - \sigma_3)/2]$  at a prescribed strain

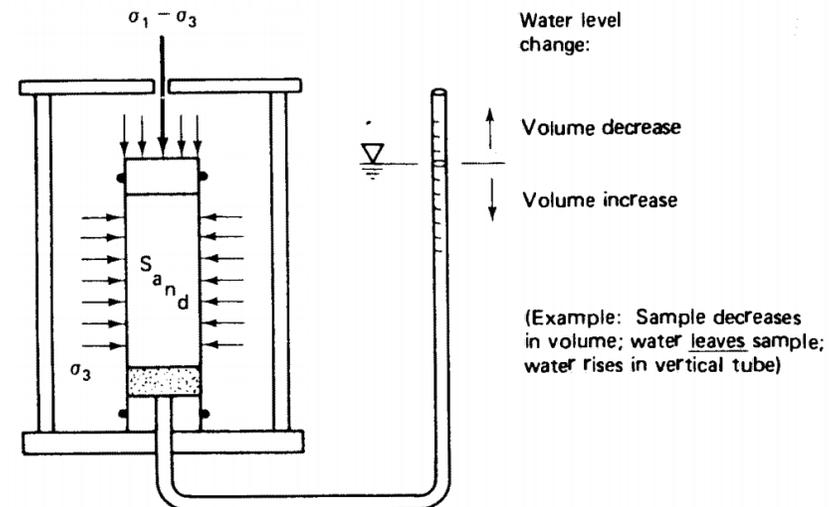
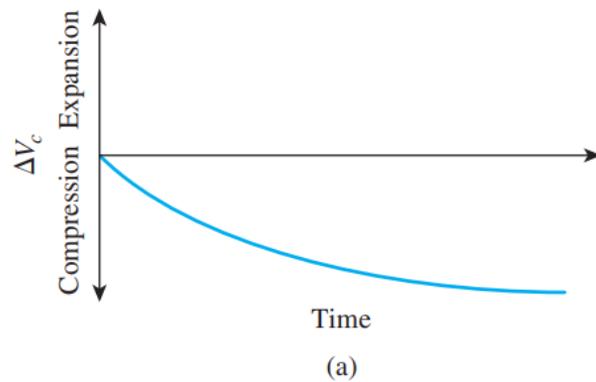
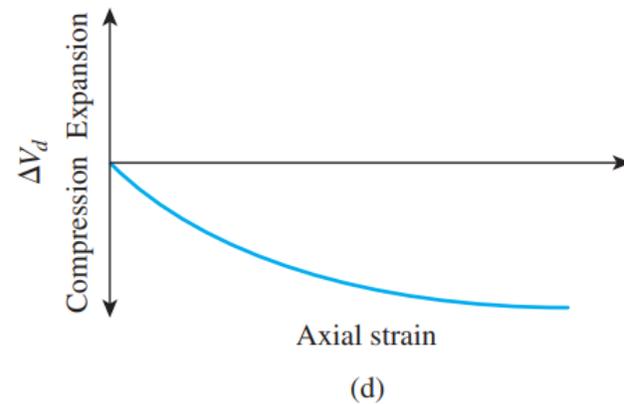
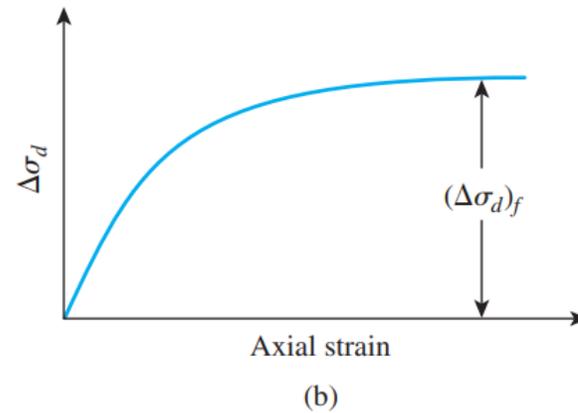


Fig. 11.2 Consolidated-drained triaxial test with volume change measurements.

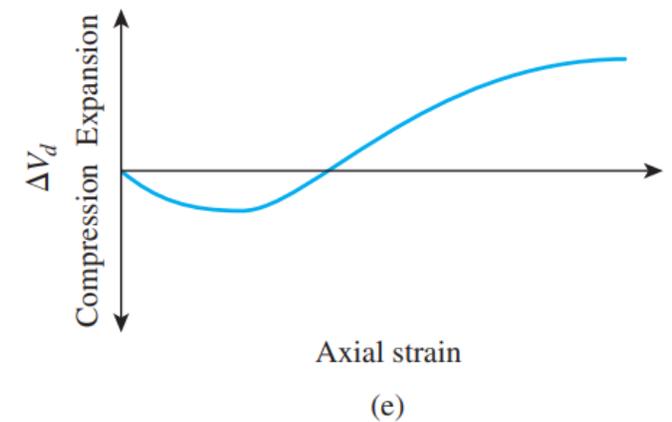
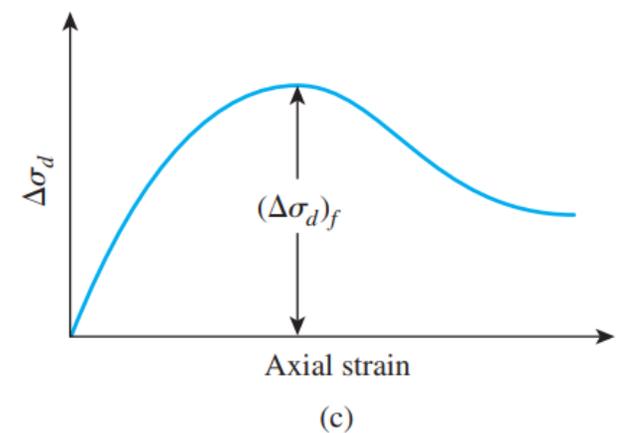
# BEHAVIOR OF SATURATED SANDS DURING DRAINED SHEAR



Result for Loose Sand

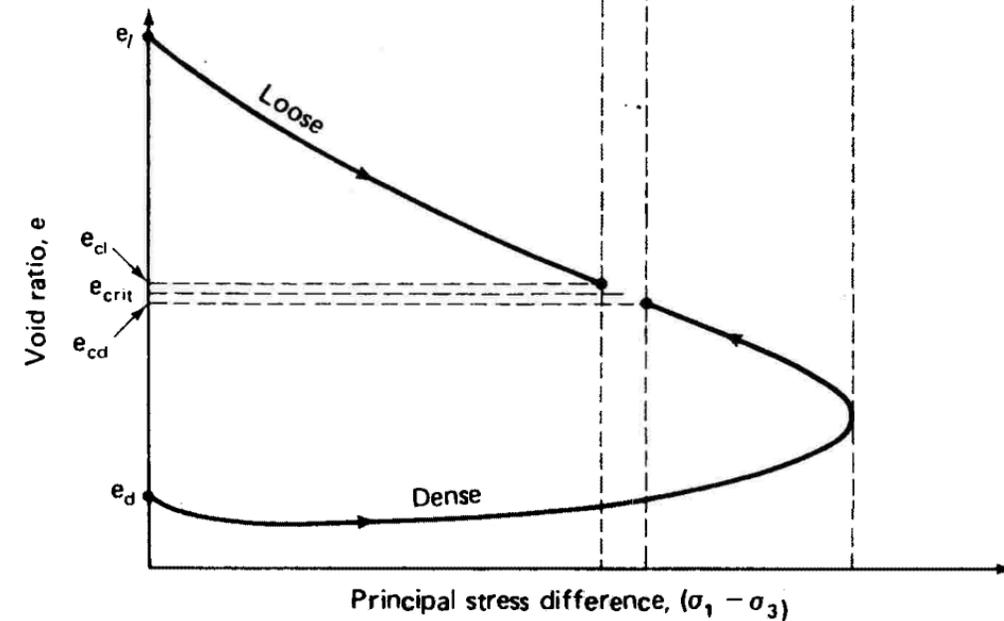
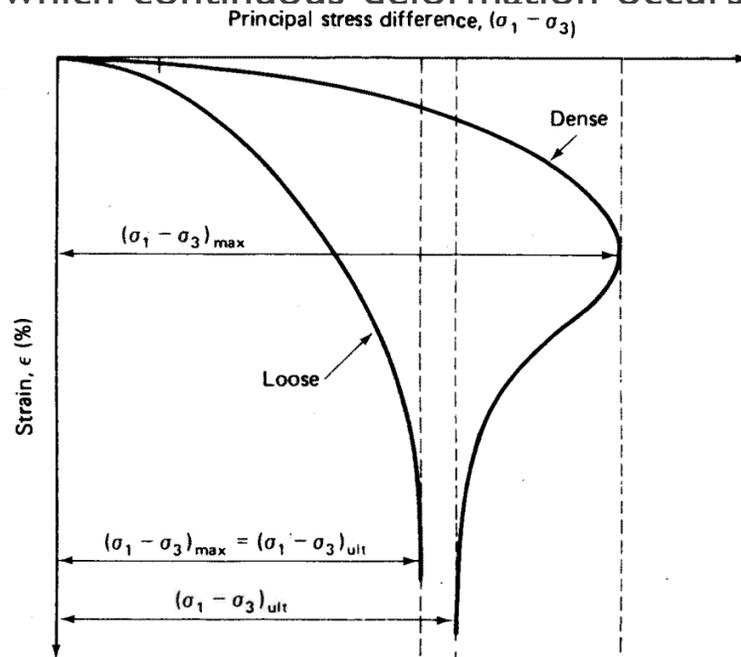


Result for Dense Sand



# BEHAVIOR OF SATURATED SANDS DURING DRAINED SHEAR

- Most of the time, we will define failure as the maximum principal stress difference, which is the same as the compressive strength of the specimen.
- When the loose sand is sheared, the **Principal Stress Difference** gradually increases to a maximum or ultimate value  $(\sigma_1 - \sigma_3)_{ult}$ . Concurrently, as the stress is increased the void ratio decreases from  $e_l$  (e-loose) down to  $e_{cl}$  ( $e_c$ -loose), which is very close to the critical void ratio  $e_{crit}$ . Casagrande (1936) called the ultimate void ratio at which continuous deformation occurs with no change in principal stress difference the critical void ratio.



# BEHAVIOR OF SATURATED SANDS DURING DRAINED SHEAR

- When the dense specimen is sheared, the principal stress difference reaches a peak or maximum, after which it decreases to a value very close to  $(\sigma_1 - \sigma_3)_{ult}$  for the loose sand. The void ratio-stress curve shows that the dense sand decreases in volume slightly at first, then expands or dilates up to  $e_{cd}$  ( $e_c$ -dense). Notice that the void ratio at failure  $e_{cd}$  is very close to  $e_{cl}$ . Theoretically, they both should be equal to the critical void ratio  $e_{crit}$ . Similarly, the values of  $(\sigma_1 - \sigma_3)_{ult}$  for both tests should be the same. The differences are usually attributed to difficulties in precise measurement of ultimate void ratios as well as non-uniform stress distributions in the test specimens.
- Evidence of this latter phenomenon is illustrated by the different ways in which the samples usually fail. The loose sample just bulges, while the dense sample often fails along a distinct plane oriented approximately  $45^\circ + \phi'/2$  from the horizontal ( $\phi'$  is, of course, the effective angle of shearing resistance of the dense sand). Note that it is at least theoretically possible to set up a sample at an initial void ratio such that the volume change at failure would be zero. This void ratio would, of course, be the critical void ratio  $e_{crit}$ .

# EFFECT OF VOID RATIO AND CONFINING PRESSURE ON VOLUME CHANGE

- We have purposely avoided defining the terms loose and dense because the volume change behavior during shear depends not only on the initial void ratio and relative density but also on the confining pressure. In this section we shall consider the effect of confining pressure on the stress-strain and volume change characteristics of sands in drained shear.
- We can assess the effect of  $\sigma_3$  (and remember, in a drained test  $\sigma_3 = \sigma_3'$ , as the excess pore water pressure is always zero) by preparing several samples at the same void ratio and testing them at different confining pressures. We would find that the shear strength increases with  $\sigma_3$ . A convenient way to plot the principal stress difference versus strain data is to normalize it by plotting the principal stress ratio  $\sigma_1 / \sigma_3$  versus strain. For a drained test, of course  $\sigma_1 / \sigma_3 = \sigma_1' / \sigma_3'$ . At failure the ratio is  $(\sigma_1' / \sigma_3')_{\max}$ .

$$\left( \frac{\sigma_1'}{\sigma_3'} \right)_{\max} = \frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2 \left( 45 + \frac{\phi'}{2} \right)$$

- Where  $\phi'$  is the effective angle of internal friction. The principal stress differences is related to the principal stress ratio by:

$$\sigma_1 - \sigma_3 = \sigma_3' \left( \frac{\sigma_1'}{\sigma_3'} - 1 \right)$$

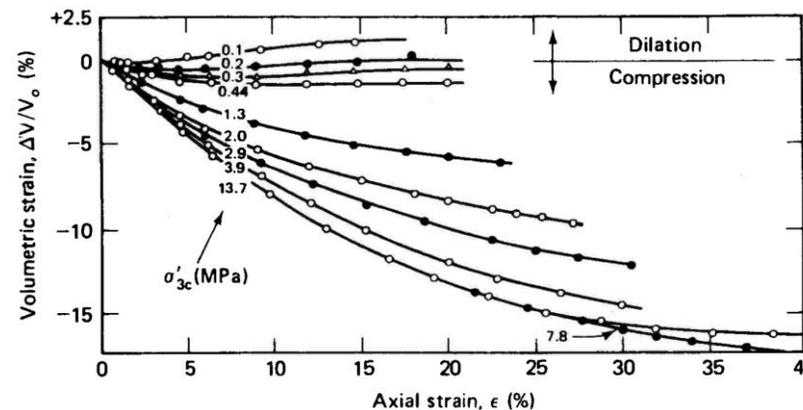
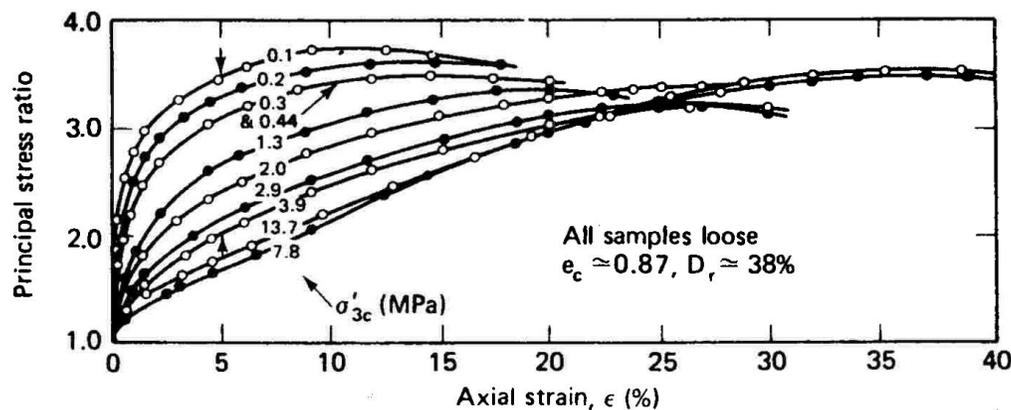
- At failure, the relationship is:  $(\sigma_1 - \sigma_3)_f = \sigma_{3f} \left[ \left( \frac{\sigma_1'}{\sigma_3'} \right)_{\max} - 1 \right]$

# EFFECT OF VOID RATIO AND CONFINING PRESSURE ON VOLUME CHANGE

- Let's look first at the behavior of loose sand. The principal stress ratio is plotted versus axial strain for different effective consolidation pressures  $\sigma'_{3c}$ . Note that none of the curves has a distinct peak, and they have a shape similar to the loose curve. The volume change data is also normalized by dividing the volume change  $\Delta V$  by the original volume  $V_o$  to obtain the volumetric strain, or,

$$\text{volumetric strain, \%} = \frac{\Delta V}{V_o} \times 100$$

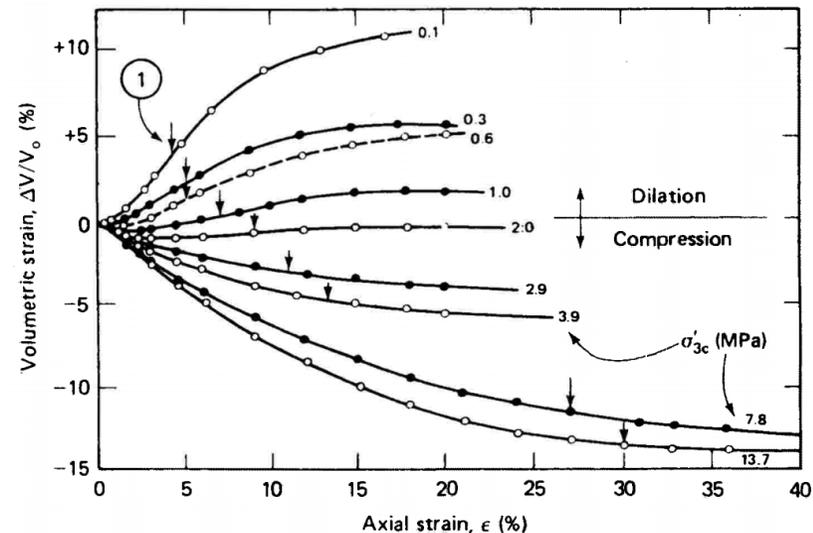
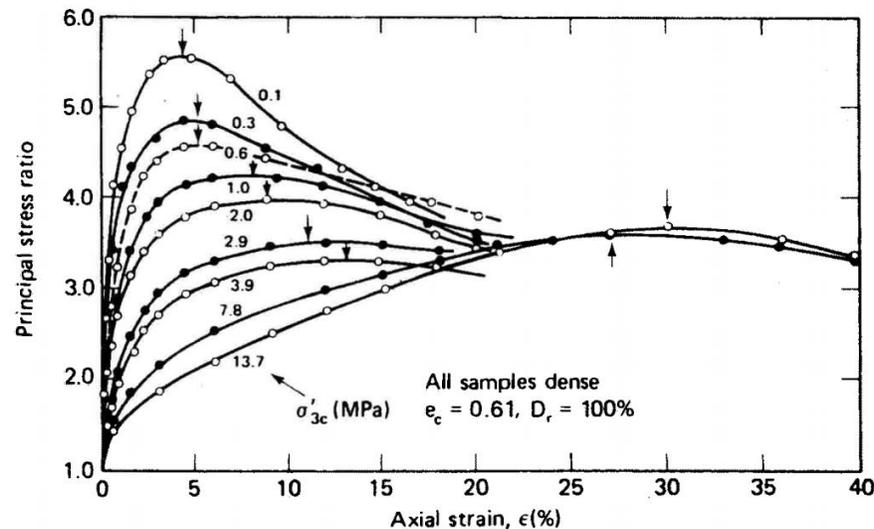
- It is interesting to look at the shapes of the volumetric strain versus axial strain curves. As the strain increases, the volumetric strain decreases for the most part. This is consistent with the behavior of a loose sand. However at low confining pressures (for example 0.1 MPa), the volumetric strain is positive or dilation is taking place. Thus even an initially loose sand behaves like a dense sand, that is, it dilates if  $\sigma'_{3c}$  is low enough.



➔ Typical triaxial test results on loose sand

# EFFECT OF VOID RATIO AND CONFINING PRESSURE ON VOLUME CHANGE

- The results of several drained triaxial tests on dense sand are presented below.



- Although the results are similar in appearance to loose sand, there are some significant differences. First, definite peaks are seen in the  $(\sigma_1'/\sigma_3')$ -strain curves, which are typical of dense sands. Second, large increases of volumetric strain (dilation) are observed. However, at higher confining pressures, dense sand exhibits the behavior of loose sand by showing a decrease in volume or compression with strain.

# EFFECT OF VOID RATIO AND CONFINING PRESSURE ON VOLUME CHANGE

- By testing samples of the same sand at the same void ratios or densities with different effective consolidation pressures, we can determine the relationship between volumetric strain at failure and void ratio or relative density.
- For drained tests, failure occurs at the same strain according to both criteria.
- It can be seen that for a given confining pressure the volumetric strain decreases (becomes more negative) as the density decreases (void ratio increases). By definition, the critical void ratio is the void at failure when the volumetric strain is zero. Thus for the various values of  $\sigma'_{3c}$ ,  $e_{crit}$  is the void ratio when  $\Delta V/V_0 = 0$ .

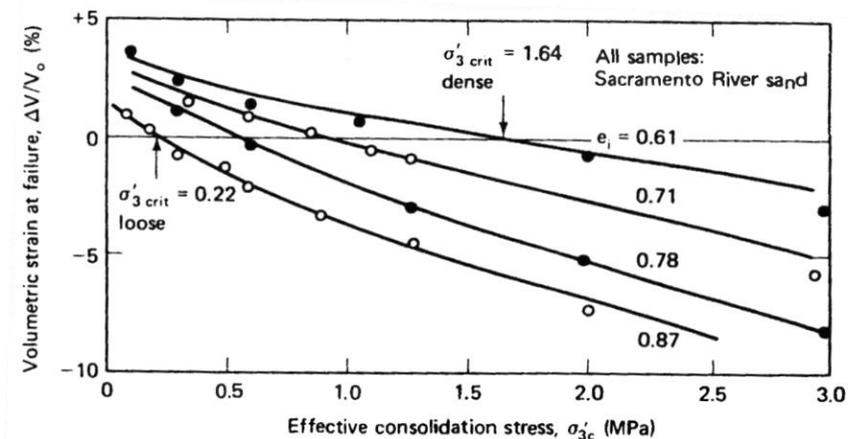
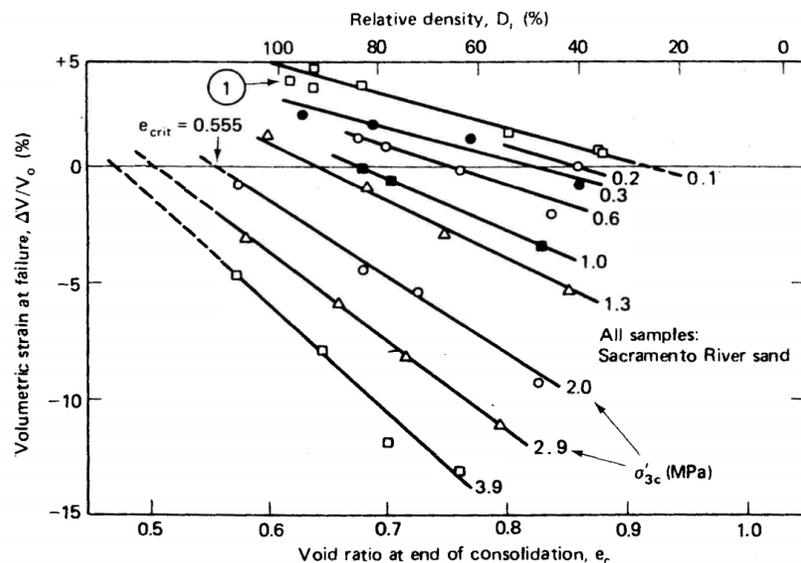


Fig. 11.8 Volumetric strain at failure versus effective consolidation stress for different initial void ratios (after Lee, 1965).



# BEHAVIOR OF SATURATED SANDS DURING UNDRAINED SHEAR

- The main difference between drained and undrained triaxial shear is that in an undrained test no volume change is allowed during axial loading. However, unless the confining pressure just happens to be at  $\sigma'_{3-crit}$ , the soil will tend to change volume during loading.
- If no tendency towards volume change takes place, then no excess pore pressure is induced. So the maximum possible pore pressure is equal to  $\sigma'_{3c} - \sigma'_{3-crit}$ .
- Since the volume change tendency is to reduce, a positive change (increase) in pore pressure is caused, which in turn results in a reduction in the effective stress. Also, if we were to run a drained test with the confining pressure equal to  $\sigma'_{3c}$ , the drained strength would be much larger than the undrained strength since its Mohr Circle must be tangent to the effective Mohr failure envelope.
- A different response occurs when we run a test with the effective confining pressure less than  $\sigma'_{3-crit}$ . Since the specimen is prevented from actually expanding, a negative pore pressure is developed which increases the effective stress. Thus as in the previous example, the limiting effective stress is the critical confining pressure  $\sigma'_{3-crit}$ .
- The whole point of this section is that we may predict the undrained behavior of sands from the drained behavior when we know the volume change tendencies.

# BEHAVIOR OF SATURATED SANDS DURING UNDRAINED SHEAR

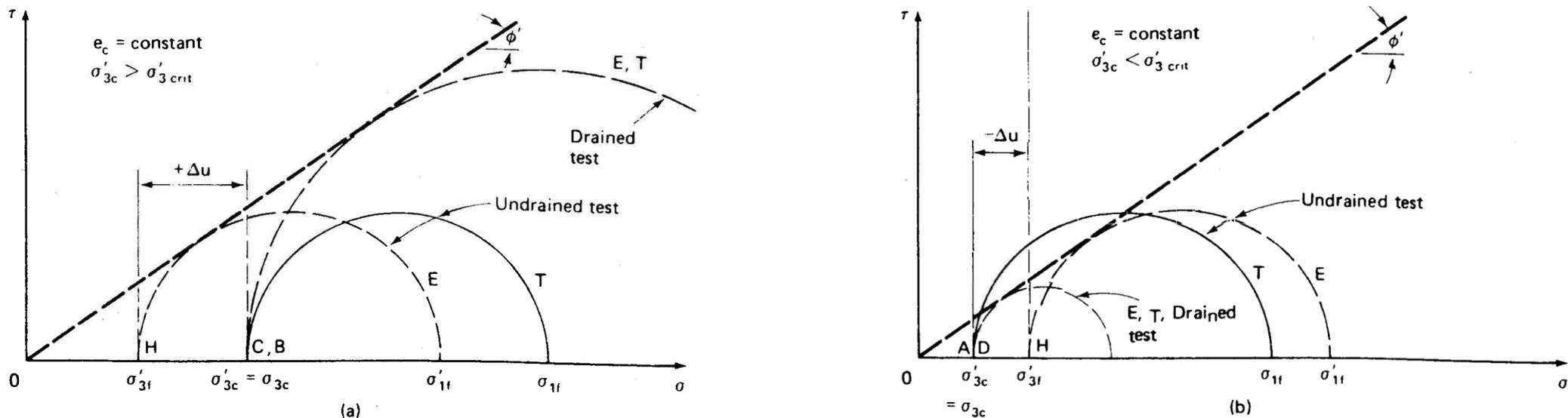


Fig. 11.11 The Mohr circles for undrained and drained triaxial compression tests: (a) case where  $\sigma'_{3c} > \sigma'_{3crit}$ ; (b) case where  $\sigma'_{3c} < \sigma'_{3crit}$ .

# BEHAVIOR OF SATURATED SANDS DURING UNDRAINED SHEAR

**TABLE 11-1** A Summary of Concepts Shown in Fig. 11.11

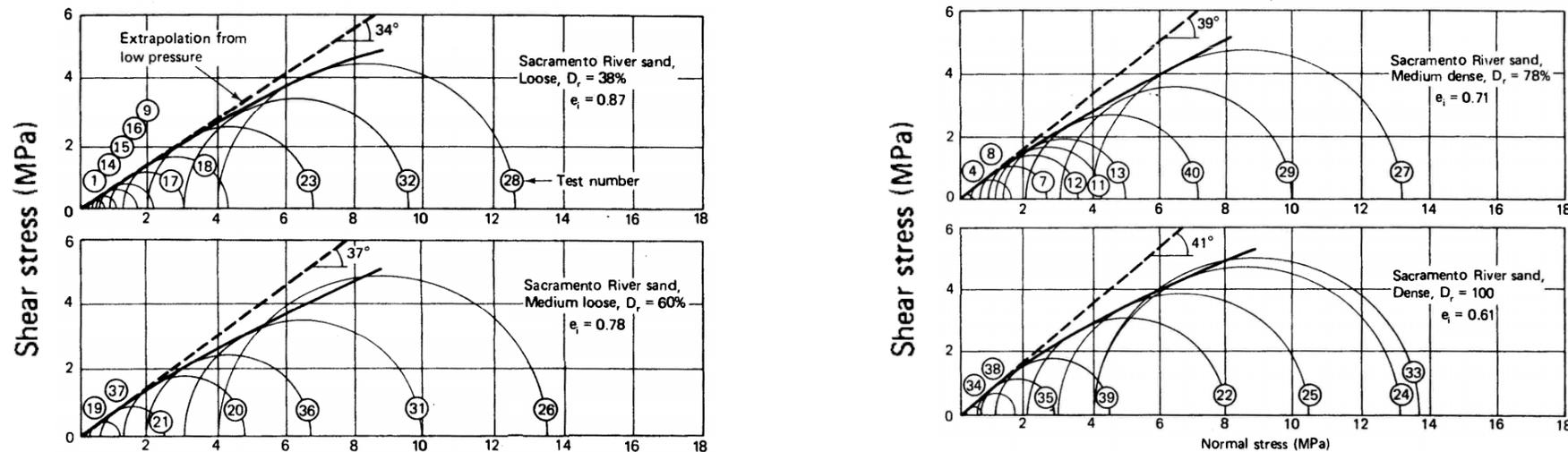
Effective Consolidation Pressure	Mohr Circles		
	Drained, Effective = Total	Undrained, Effective	Undrained, Total
$\sigma'_{3c} > \sigma'_{3 \text{ crit}}$	Larger than undrained	Smaller than drained: Left of total stress circle $\sigma'_{3f} < \sigma'_{3c}$	Smaller than drained: Right of effective stress circle
$\sigma'_{3c} < \sigma'_{3 \text{ crit}}$	Smaller than undrained	Larger than drained: Right of total stress circle $\sigma'_{3f} > \sigma'_{3c}$	Larger than drained: Left of effective stress circle
$\sigma'_{3c} \equiv \sigma'_{3 \text{ crit}}$	All circles would be the same; because no volume change tendencies exist, $\Delta u = 0$ during the test.		

# FACTORS THAT AFFECT THE SHEAR STRENGTH OF SANDS

- Since sand is a “frictional” material we would expect those factors that increase the frictional resistance of sand to lead to increases in the angle of internal friction. First, let us summarize the factors that influence  $\phi$ .
  1. Void ratio or relative density
  2. Particle shape
  3. Grain size distribution
  4. Particle surface roughness
  5. Water
  6. Intermediate principal stress
  7. Particle size
  8. Over-consolidation or prestress
  
- Void ratio, related to the density of the sand, is perhaps the most important single parameter that affects the shear strength of sands. Generally speaking for drained tests either in the direct shear or triaxial test apparatus, the lower the void ratio, the higher the shear strength.

# FACTORS THAT AFFECT THE SHEAR STRENGTH OF SANDS

- The Mohr circles for the triaxial test data presented earlier are shown below for various confining pressures and for initial void ratios.



**Fig. 11.12 Mohr circles and failure envelopes from drained triaxial tests, illustrating the effects of void ratio or relative density on shear strength (after Lee, 1965; also after Lee and Seed, 1967).**

- You can see that as the void ratio decrease, or the density increases, the angle of internal friction or angle of shearing resistance  $\phi$  increases.
- Another thing you should notice is that the Mohr failure envelopes are curved, that is  $\phi'$  is not constant if the range in confining pressures is large.

# FACTORS THAT AFFECT THE SHEAR STRENGTH OF SANDS

- The final factor on our list, overconsolidation or prestress of sands, has been found to not significantly affect  $\phi$ , but it strongly affects the compression modulus of granular materials. Ladd et al (1977) discuss the various effects of prestress on behavior of granular materials.
- All the factors mentioned before are summarized below.

**TABLE 11-3 Summary of Factors Affecting  $\phi$**

Factor	Effect
Void ratio $e$	$e \uparrow, \phi \downarrow$
Angularity $A$	$A \uparrow, \phi \uparrow$
Grain size distribution	$C_u \uparrow, \phi \uparrow$
Surface roughness $R$	$R \uparrow, \phi \uparrow$
Water $W$	$W \uparrow, \phi \downarrow$ slightly
Particle size $S$	No effect (with constant $e$ )
Intermediate principal stress	$\phi_{ps} \geq \phi_{ix}$ (see Eqs. 11-5a, b)
Overconsolidation or prestress	Little effect

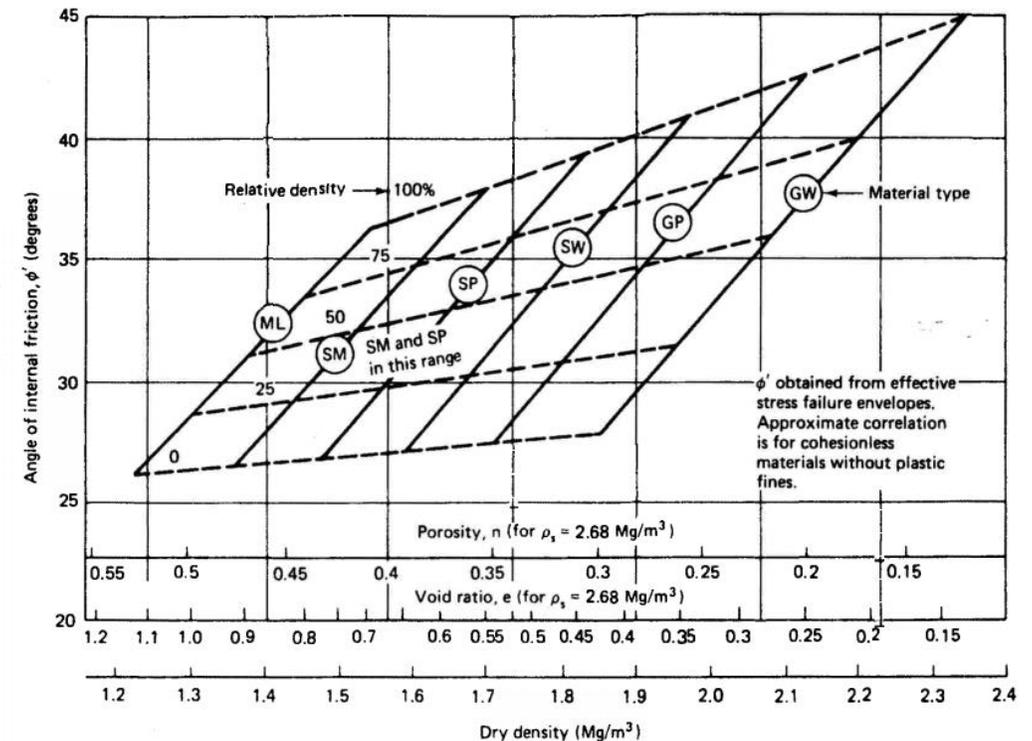


Fig. 11.13 Correlations between the effective friction angle in triaxial compression and the dry density, relative density, and soil classification (after U.S. Navy, 1971).

# THE COEFFICIENT OF EARTH PRESSURE AT REST FOR SANDS

- The best known equation for estimating  $K_o$  was derived by Jaky (1944, 1948) which is a theoretical relationship between  $K_o$  and the angle of internal friction  $\phi'$ , or:

$$K_o = 1 - \sin \phi$$

- Schmidt (1966, 1967) and Alpan (1967) suggested that the increase in  $K_o$  could be related to the over-consolidation ratio (OCR) by

$$K_{o-oc} = K_{o-nc} (\text{OCR})^h$$

- Where  $h$  = an empirical exponent
- Values of  $h$  range between 0.4 and 0.5 and even as high as 0.6 for very dense sands.
- Ladd (1977) pointed out that this exponent itself varies with OCR, and it seems to depend on the direction of the applied stresses. For example, Al-Hussaini and Townsend (1975) found a significantly lower  $K_o$  during reloading than during unloading in laboratory tests on a uniform medium sand. Thus  $K_o$  appears to be very sensitive to the precise stress history of the deposit.

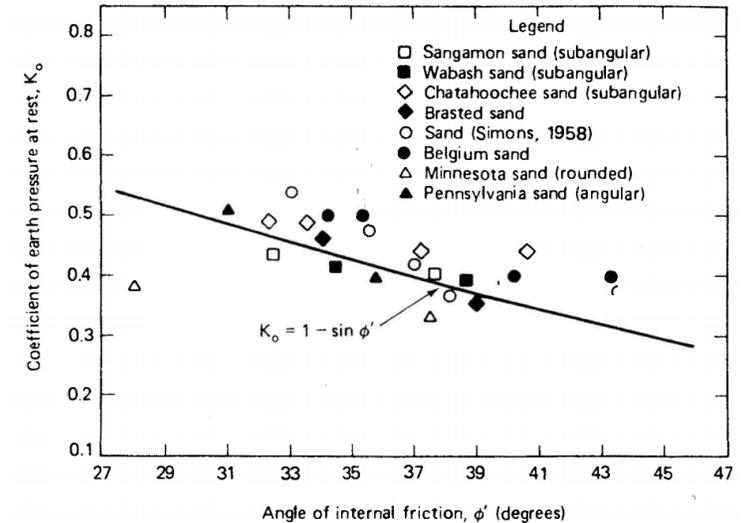


Fig. 11.14 Relationship between  $K_o$  and  $\phi'$  for normally consolidated sands (after Al-Hussaini and Townsend, 1975).

# LIQUEFACTION AND CYCLIC MOBILITY BEHAVIOR OF SATURATED SANDS

- Castro (1969) presented the results of three CU tests and one CD test, all hydrostatically consolidated to 400 kPa. The relative densities  $D_r$  of each specimen after consolidation are also indicated on the figure next to the stress-strain curve for each specimen. The specimens were loaded axially (monotonically) by small dead-load increments of weight applied about every minute to the soil sample.
- In the test A (the lowest  $D_r$ ), the peak stress difference of 200 kPa was reached in 15 min, which corresponded to an axial strain of about 1%. Then, when the next small increment of load was applied, the specimen suddenly collapsed – liquefied – and in about 0.2s the stress decreased from 200 to 30 kPa at 5% strain, where it remained as the specimen continued to flow.
- Notice how the pore pressure for specimen A remained the same during flow. At this maximum value of pore pressure, the effective minor principal stress was only about 15 kPa, and if you calculate the  $\phi'$  from this stresses, you get  $\phi' = 30^\circ$ .

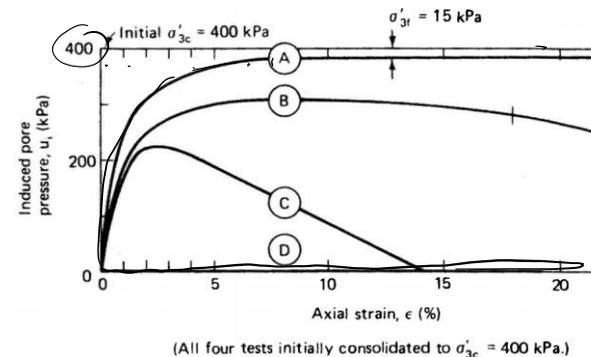
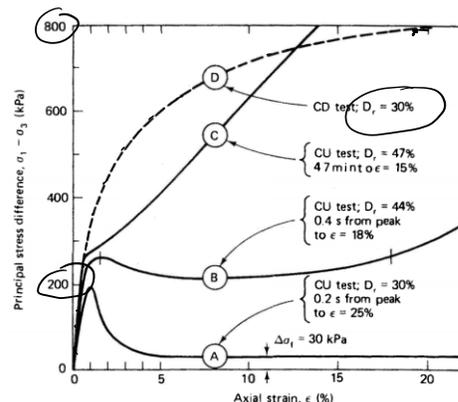


Fig. 11.16 Comparison of three hydrostatically consolidated CU tests and one CD test on banding sand loaded incrementally to failure (after Casagrande, 1975, from Castro, 1969).

# LIQUEFACTION AND CYCLIC MOBILITY BEHAVIOR OF SATURATED SANDS

- The total and effective Mohr circles at the peak and during flow after liquefaction are shown in Figure below. Also shown for comparison are the results of the CD test on the same sand at the same  $D_r$ . Both tests indicate that  $\phi=30^\circ$  for this loose sand, although as pointed out by Casagrande (1975), the agreement may be only a coincidence. In any event, the effective stress circle at the peak on maximum stress difference lies below the effective failure envelope.

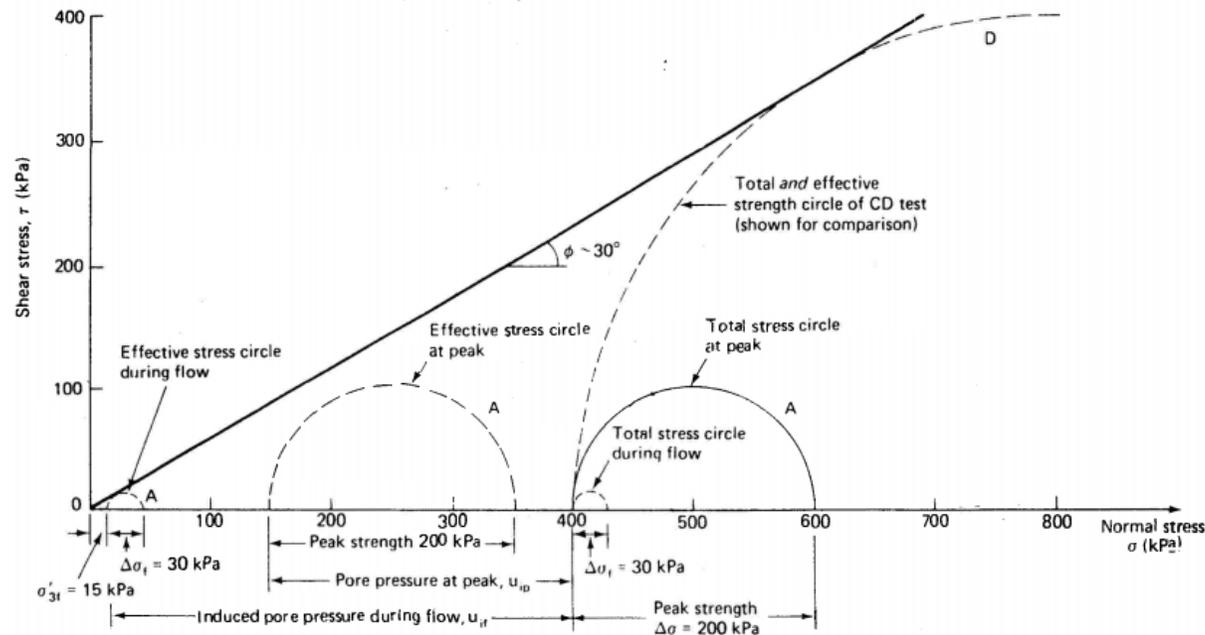


Fig. 11.17 Mohr circles in terms of total and effective stresses for the CU test (specimen A) and the CD test of Fig. 11.16. Condition at both maximum stress difference and during flow are shown (after Casagrande, 1975).

- Figure beside is another good illustration of the very large differences in the strength of sands, depending on the drainage conditions.
- Here you see the results of CD versus CU tests on the same sand at the same relative density and at the same effective consolidation stress.
- The differences are even greater when you consider the strength of the sand after liquefaction. In a flow slide, this sand would simply flow out like a very dense liquid, and its equilibrium slope angle might be only a very few degrees.

# LIQUEFACTION AND CYCLIC MOBILITY BEHAVIOR OF SATURATED SANDS

## ■ Behavior of Loose sand under Cyclic loading

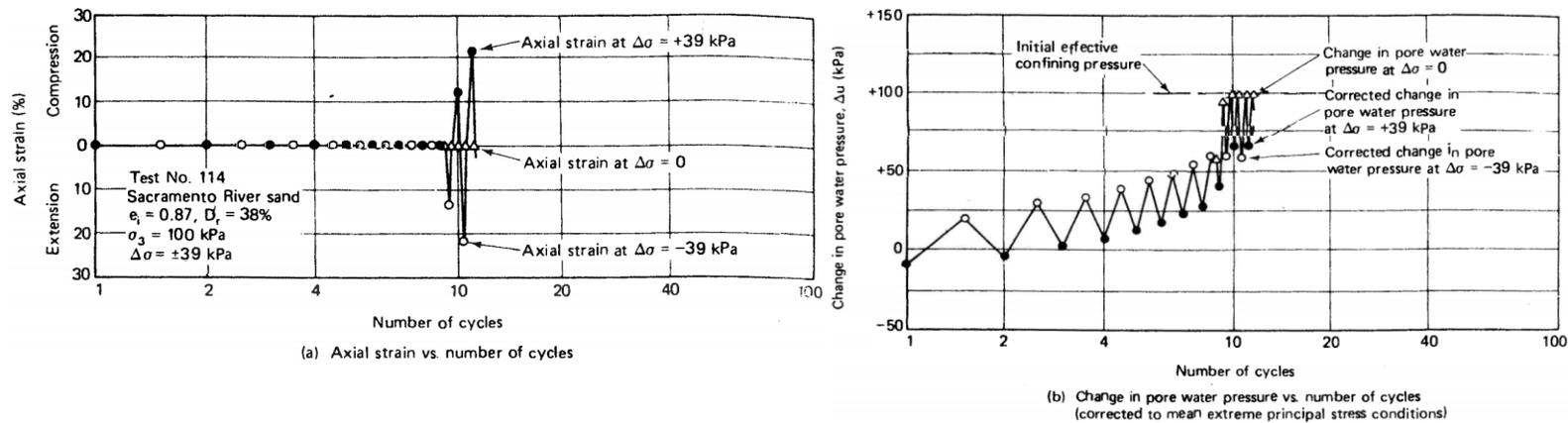


Fig. 11.18 Typical cyclic triaxial test on loose sand (after Seed and Lee, 1966).

## ■ Behavior of Loose sand under Cyclic loading

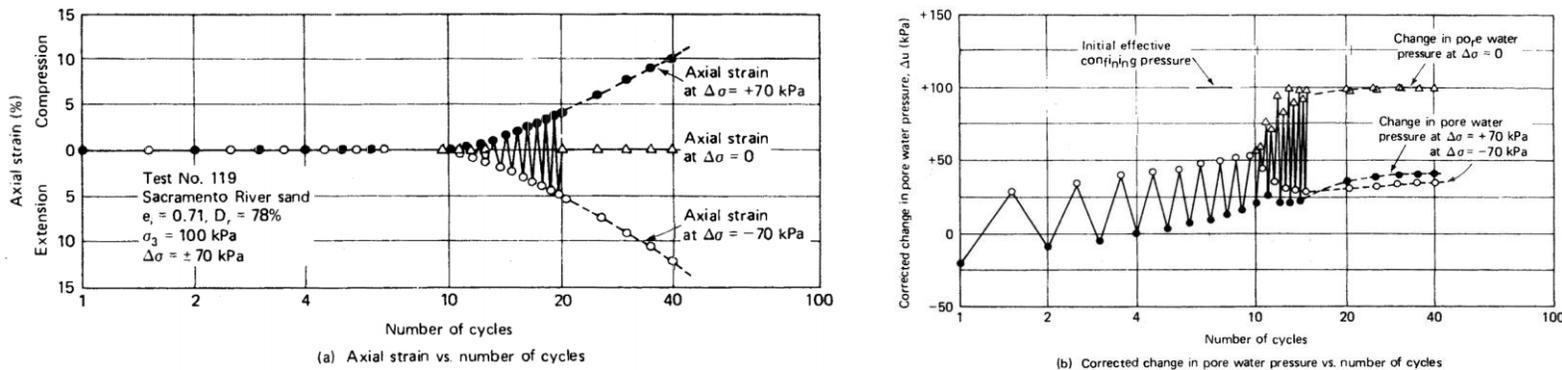
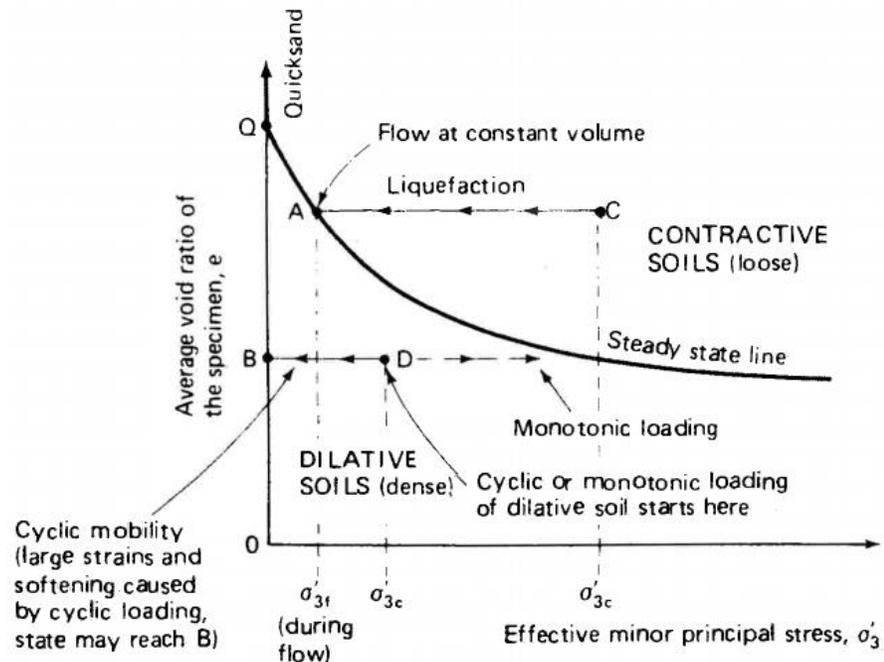


Fig. 11.19 Typical cyclic triaxial test on dense sand (after Seed and Lee, 1966).

# LIQUEFACTION AND CYCLIC MOBILITY BEHAVIOR OF SATURATED SANDS

- Work by Castro (1975) explained that we were seeing two basically different phenomena
  1. Classical liquefaction of loose sands
  2. The phenomenon called cyclic mobility which occurs in the laboratory during cyclic triaxial or simple shear tests.
- These two phenomena are illustrated in Figure below.



- For example, a sample starting at point C when stressed or vibrated develops a large amount of positive excess pore pressure and ends up point A on the steady state line, where the sample has no further tendency to change volume.
- On the other hand, a dense dilative specimen originally at point D below the steady-state line, if subjected to cyclic shear, will move towards point B, a condition of zero effective stress. This is the condition of cyclic mobility.
- If the same sample were loaded monotonically or statically in an ordinary triaxial test, then it would go in the opposite direction towards the steady-state line.

# REFERENCE

The following publications can be referred for a more detailed analysis and understanding.

1. Abramson, L.E, Lee, T.S, Shama, S., and Boyce, G.M, (1996), Chapter 5 Laboratory Testing and Interpretation, John Wiley and Sons, Inc.
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4. Ladd, C. C. (1971). Strength Parameters and Stress – Strain Behavior of Saturated Clays, Research Report R71-23, Soils Publication 278, Massachusetts Institute of Technology.